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The structure of prices between the futures and cash markets for 90-day Treasury bills

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CHAPTER I. INTRODUCTION

On January 6, 1976, the International Monetary Market (IMM) of the Chicago Mercantile Exchange initiated trading in futures contracts for the delivery of U.S. Treasury bills. Until that time, futures markets had traditionally been associated with the intertemporal exchange of agricultural commodities. With the introduction of Treasury futures, however, the concepts of commodity futures trading have been applied to one of the most basic of all commodities - money.

Like all other commodities, money has a price - the interest rate which must be paid for its use. In theory, supply and demand for money ultimately determines its price. However, unlike most commodities which are supplied by numerous individuals or firms, the supply of money is basically determined by the Federal Reserve System. As long as the demand for money is relatively stable and the supply of money is permitted to increase in accordance with the financial needs of a growing economy, the price of money should not vary substantially over time. In fact, historical time series of interest rates show that until recent years, interest rate stability was fairly typical of the U.S. economy.

Over the last 10 to 15 years, however, money and capital markets have experienced a much greater variability in the prices of financial securities traded in these markets. This is due, in part, to a seemingly insatiable demand for money and credit expansion in both the private and public sectors of the economy. The

Federal Reserve System is faced with a difficult choice. If the money supply is allowed to grow proportionally to the surge in demand, inflation may be exacerbated. On the other hand, if the Federal Reserve System retards the growth of the monetary aggregates, then interest rates are likely to rise. The difficulty lies in the fact that it is not possible to control both interest rates and the stock of money simultaneously.

As the goals of the Federal Reserve System have changed and as its ability to control the money stock has come under question, it has become clear that interest rates have become more volatile now than at any time in the past. Evidence of the volatility of short term rates is illustrated in Figure 1.1 which shows a twenty-five year time series in 90-day Treasury bill rates. As a result of this interest rate volatility, financial institutions whose major assets and liabilities are composed of interest-sensitive securities have been subject to increased uncertainty with respect to the value of those securities. Unstable interest rates have been the primary motivation for the development and subsequent growth of Treasury futures markets.

With the existence of these markets, financial institutions have the opportunity to hedge the market risk of their cash transactions by taking positions in the futures markets opposite to those in the cash market. At the same time, the futures market also afford speculators the opportunity to profit from the risks of changing interest rates. The concepts of hedging and speculation have

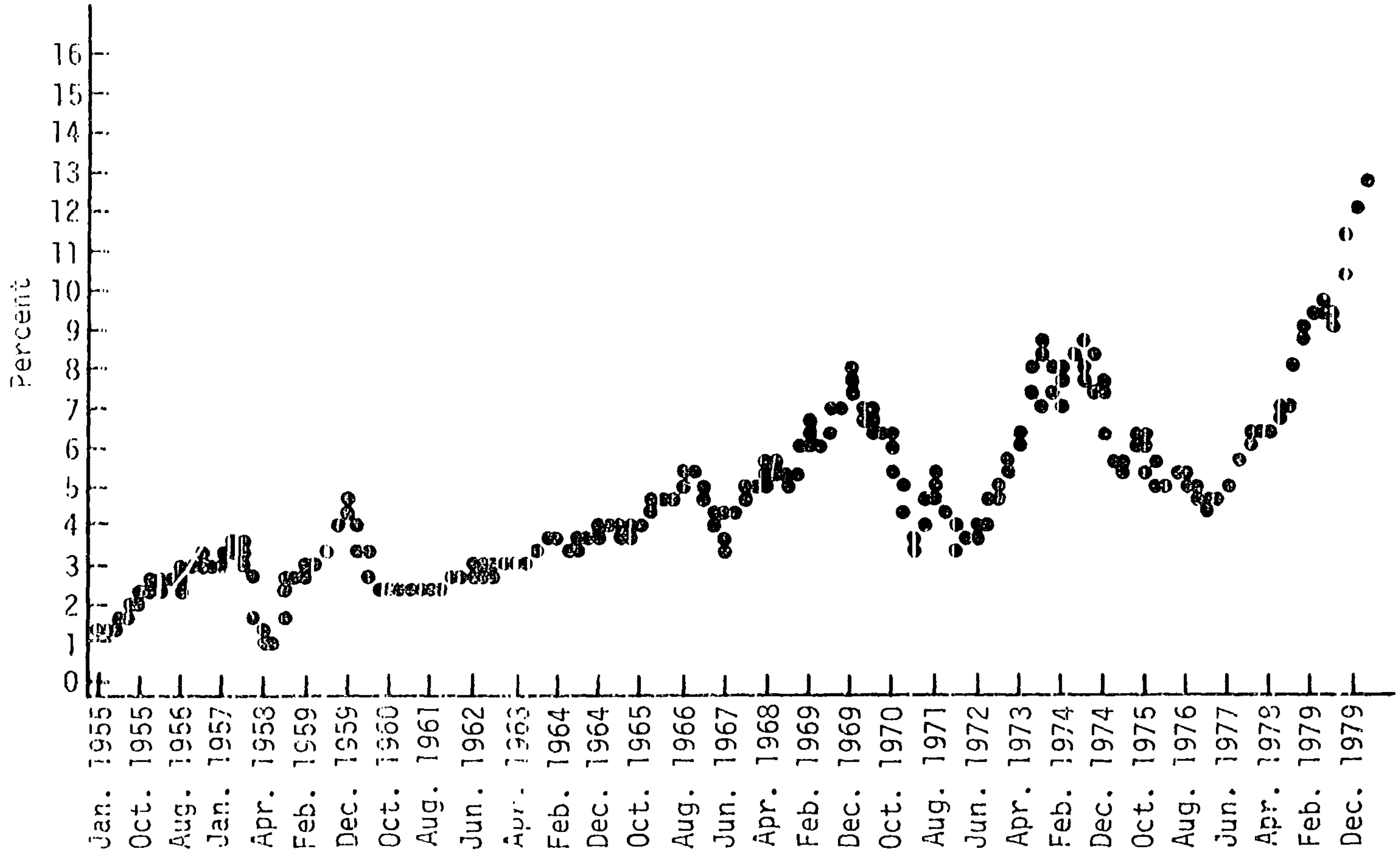

Figure 1.1. Volatility of the 90-day Treasury bill (new issue) rate over time, 1955-1980 (U.S. Department of Commerce, Bureau of Economic Analysis, Survey of Current Business)

been recognized for many years with respect to other commodities which are characterized by volatile prices. Thus, it was only natural that Treasury securities should be treated in the same fashion once the need became apparent. Interest rate volatility creates interest rate risk and the existence of Treasury futures markets provides the opportunity to manage that risk, whether it be to reduce it or profit from it.

Futures contracts for 90-day Treasury bills were first traded on the IMM and subsequently, similar contracts were introduced and are currently trading on the Amex Commodities Exchange, Inc. (ACE), an affiliate of the American Stock Exchange and on the Commodity Exchange, Inc. (Comex). The New York Futures Exchange (NYFE), a wholly owned subsidiary of the New York Stock Exchange, plans to begin trading in Treasury futures in the spring, 1980.

The 90-day Treasury bill futures market has been widely used and the number of market participants has expanded greatly. Among the active participants are commercial banks, savings and loan associations, securities dealers, mortgage bankers, pension funds and financial corporations in addition to the futures industry itself, which consists of commodity pools and funds as well as individual traders. The rapid growth in the 90-day Treasury bill futures market is illustrated in Table 1.1, which shows the steady increase in the open interest of 90-day bill futures. Open interest is the total number of futures contracts of a given commodity which have not yet been offset by opposite futures transactions nor fulfilled by

Table 1.1. Open Interest of 90-day Treasury Bills (CFTC permanent records)

Date		Open Interest[a]	Date		Open Interest[b]
January	1976	358	January	1978	12,392
February	1976	586	February	1978	13,645
March	1976	890	March	1978	15,585
April	1976	1,278	April	1978	17,583
May	1976	1,424	May	1978	20,283
June	1976	1,529	June	1978	23,684
July	1976	1,721	July	1978	27,548
August	1976	2,015	August	1978	34,580
September	1976	1,928	September	1978	33,956
October	1976	2,031	October	1978	48,024
November	1976	2,333	November	1978	52,500
December	1976	3,081	December	1978	58,985
January	1977	2,688	January	1979	53,691
February	1977	3,055	February	1979	54,172
March	1977	4,108	March	1979	52,567
April	1977	5,239	April	1979	56,355
May	1977	6,403	May	1979	63,238
June	1977	7,055	June	1979	60,962
July	1977	8,668	July	1979	46,667
August	1977	10,430	August	1979	38,917
September	1977	10,726	September	1979	41,399
October	1977	12,292	October	1979	35,377
November	1977	14,145	November	1979	42,690
December	1977	16,902	December	1979	35,945

[a]Average monthly open interest.

[b]Month end open interest.

delivery of the commodity.

Benefits and Costs

With the increase in activity and volume in the 90-day Treasury bill futures markets, the question arises as to whether or not this market provides any net social benefits or costs to society. In the case of agricultural commodities, it has been well-documented that there exist certain public benefits that arise as a result of trading in commodity futures. For example, it is generally accepted that one of the most important benefits of trading grain futures is the price discovery signals that the futures market provides regarding the storage of grain over time. In the aggregate, the array of grain futures prices extant at a particular point in time provides spot market participants with valuable information regarding how much grain to store and when to store it. Thus, the prices provided by the grain futures market are crucial to the orderly flow and distribution of grain over time.

To assess the benefits or costs of trading Treasury futures, it is essential to determine how the futures market is interrelated with the cash market. After all, the benefits or costs which result from an active futures market will in some manner be transmitted to the cash market. Therefore, in order to determine the effects of futures trading, it is necessary to examine the structural relationship between the two markets. Because most of the information produced by a futures market is embodied in prices, the relationship of prices between the futures and cash markets will be analyzed. This will be important for two reasons: to determine the value of information produced in the futures market and secondly, to assess the effects that futures trading has on the stability of prices in the cash market.

Value of information

Interest rate expectations form an important part of economic and business planning and they enter into both micro and macroeconomic models at many different levels. Because interest rate expectations are so widely used, the accuracy of their source is important. Estimates of expected interest rates may be obtained from private surveys or they can be estimated from the current term structure of interest rates. With the commencement of futures trading in Treasury securities, yet another source of interest rate expectations is now available.

It would seem plausible that interest rate expectations derived from each of the above sources should be consistent with one another over time. Therefore, the question arises whether the value of information supplied by the futures market is diminished if futures rates are not consistent with the information derived from other sources, for example, the term structure of interest rates. The consistency between these two sources of information may be determined by analyzing the price structure between the futures market and the cash market. If it can be shown that futures prices and cash prices are structured in such a way that the futures rates are consistent (not necessarily identical) with expected rates derived from the cash market, then the futures market for 90-day Treasury bills will prove to be a valuable and inexpensive source of information on interest rate expectations.

Stability of cash prices

An analysis of the futures-cash price structure is also important to assess the potential costs that trading in Treasury bill futures market may have on the cash market. If the futures contract is priced efficiently with respect to the cash market, then there will exist a futures-cash price link through which aberrations occurring in the futures market will be transmitted to the cash market. For example, the Treasury and Federal Reserve System share the concern that destabilizing speculation or futures market squeezes will have disruptive effects on prices in the cash market. If futures market activities create more volatility in the cash prices of Treasury bills, then these securities may exhibit less liquidity and the Treasury Department may find it more difficult to market them. As a consequence, the Treasury's ability to manage the Nation's debt may be impaired, making it more expensive to finance public programs.

Whether it is concluded that the futures market for Treasury bills produces net benefits or net costs will ultimately depend on a thorough examination of the futures-cash price structure. The issue of the price structure has been addressed by a number of authors. Although most of the previous work has been conducted to evaluate the information produced in the futures market, the analysis is just as relevant with respect to the effects of futures trading on the cash market.

Much of the analysis regarding the price structure has been concerned with testing for the efficiency of the futures market. Most of the tests postulate that if the futures market is efficient, then the futures rate will be equal to the forward rate implied in the term structure. If the two are identical, the conclusion then follows that the estimates of interest rate expectations produced in the futures market are consistent with those in the cash market. Other studies have set out to test for the validity of the pure expectations hypothesis which says that forward rates implied in the term structure are unbiased estimates of futures spot rates. This approach assumes that futures rates are unbiased estimates of future spot rates and thus, these tests also compare forward rates with futures rates. According to these tests, the validity of the pure expectations hypothesis is supported if it is found that the forward and futures rates are identical over time.

Most of the theories which concern the price structure between the futures and cash market conclude at one point or another that the forward rate should equal the futures rate. In this instance, expectations produced in the futures market are considered to be consistent with those in the cash market. Under the above circumstances, the value of information produced by the futures market is confirmed and the net benefits resulting from futures trading substantiated. Many authors have found, however, that the futures rates and the forward rates are not identical over time. From this, they conclude that the futures contract is not priced consistently with respect to the cash market and therefore, the futures market is inefficient. Based on the hypotheses tested, these findings would seem to cast doubt on the contention that the information produced in the Treasury bill futures market is valuable and consistent with similar information produced in the cash market.

Purpose of Study

The purpose of this study is to reevaluate the futures cash price structure taking into account various characteristics of the markets that have been overlooked by previous authors. For example, most of the authors have implicitly assumed that futures contracts are economically equivalent to forward contracts and on this basis, they have tested for market efficiency by comparing futures rates to forward rates. There are, however, important differences which distinguish futures contracts from forward contracts. Although a full discussion of their characteristics is deferred until Chapter II, the differences between the two types of contracts have significant implications for the analysis of the futures-cash price structure.

Specifically, this study examines the relationship of prices in the futures market to those in the cash market giving explicit consideration to all aspects which distinguish futures contracts from forward contracts based on cash Treasury bills. These include commissions, initial margins, maintenance margins, expectations, and uncertainty. Each of these considerations is included in a model of the equilibrium price of a futures contract. Several hypotheses result from the model. The first states that the observed rate on a futures contract is composed of a risk free rate, transactions costs and a risk premium. Only the risk adjusted component of the observed futures rate is identical to the forward rate implied in the term structure. The second hypothesis, which follows from the first, states that even in an efficient market, the forward rate will not be identical to the observed futures rate if the variables which distinguish a futures from a forward contract are significant. Inequality between the rates does not necessarily imply that the two markets are inconsistent with respect to one another.

The theory suggests that tests of efficiency which compare the futures rate with the forward rate are inappropriate, as is the conclusion that inequality between the two sets of rates implies market inefficiency. On the contrary, the implication of the following analysis is that even in an efficient market, the futures rate will not be identical to the forward rate. The relationship between these two variables is an empirical matter, which will depend on the significance of various characteristics which differentiate futures contracts from forward contracts. The nature of the bias between the futures and forward rates will be explored econometricallv on the basis of the transactions costs and a risk premium.

To introduce many of the concepts that are frequently used in the forthcoming analysis, Chapter II is devoted to providing a full description of the markets in which the various contracts are traded. In addition to this, the contracts themselves will be described in detail along with the major differences between futures contracts and forward contracts. Having outlined the institutional details of the contracts and their respective markets, the emphasis will then turn to previous studies of the Treasury bill futures market. The focus of Chapter III, the review of literature, will move from the general to the specific, beginning with a comprehensive study of all Treasury futures markets and moving on to more detailed studies which concentrate solely on the price structure and efficiency of the 90-day Treasury bill futures market. When the highlights and deficiencies of these studies have been explored, an alternative theory of the equilibrium futures price will be developed in Chapter IV. The qualitative aspects and implications of this equilibrium price will be analyzed, especially with regard to the factors which make futures contracts distinct from forward contracts. Following the qualitative development of the equilibrium futures price and the relationship between the futures rate and the forward rate, the empirical analysis of Chapter V will test quantitatively the significance of the behavioral relationships developed in the theoretical analysis. The empirical findings will include various tables and graphs representing the observed price relationships over time and the econometric results attempting to explain them. Finally, Chapter VI will present the major conclusions of this inquiry.

CHAPTER II. THE MARKETPLACE: SPOT, FUTURES AND FORWARD

To fully understand analyses of the price structure between the Treasury bill futures market and the cash market, it is necessary to understand the institutional details and basic concepts of the marketplace in which these financial securities are traded. To this end, a description of the spot, futures and forward markets for Treasury securities is presented, followed by an introduction to the concepts of market efficiency and arbitrage.

The Treasury Bill Cash Market

The Treasury bill cash market constitutes the largest component of the U.S. money market. As obligations of the United States Treasury, these securities are backed by the full faith and credit of the U.S. Government. As such, they are of the highest quality, and the risk associated with their default is minimal. Bills are issued with maturities of 3, 6 and 12 months and are sold in minimum denominations of $10,000 and multiples of $5,000 above the minimum. They are issued in book-entry form meaning that the purchaser receives a receipt, rather than an engraved certificate, as evidence of ownership.

The primary and secondary market

Treasury bills are traded in both a primary market and a secondary market. The primary market is a regular weekly auction for 91- and 182-day bills. Each Tuesday, the size of the following week's offering of Treasury bills is announced. On the following Monday, the 91- and 182-day securities are auctioned and payment and issuance occur on the Thursday thereafter. One-year Treasury bills are auctioned once each month. The primary market is composed mainly of large banks and recognized government security dealers, both of which are permitted to submit tenders at the auction for the accounts of their customers as well as their own accounts. Other bidders may submit tenders, but for their own accounts only. The primary market is basically an "insider" market where the Treasury auctions its weekly debt to those institutions who will in turn market and distribute the Treasury securities in the secondary market.

The secondary market for Treasury bills is a dealer-made market in which enormous quantities of bills are traded under very competitive conditions at very small margins. It is not uncommon for the daily volume of trading in the secondary market to reach as high as $3 billion to $5 billion. Due to the size of this market and the fact that government security dealers are obliged to make a continuous market, Treasury bills can be liquidated at any time.

Treasury bills are priced on a discount basis with the rate of return being defined implicitly by the difference between the face value and the purchase price of the bill. For example, an investor who bids 98.50 on a 3-month Treasury bill is offering to pay $98.50 per $100 of face value on whatever quantity of bills he bids for. In this case, the discount is $1.50 over three months and the implicit rate of return is approximately 6.09 percent compounded quarterly. Since the secondary market for Treasury bills is a dealer-made market, bills are purchased at the dealer's asked price and sold at the dealters bid price. This bid-ask spread constitutes the transactions costs of doing business in the cash market. It also represents the profit margin received by the dealer in return for his role as market-maker.

The major participants in the Treasury bill secondary market include the government security dealers and their retail trade which is comprised of banks, thrift institutions, corporations, pension funds, and a numerous array of other financial and nonfinancial institutions. In conjunction with one another, these market participants make up what is one of the largest and most competive markets in the world.

The delivery vehicle on the futures contract

Treasury bills have certain characteristics that make them distinct among money market instruments and suitable as the delivery vehicle on the Treasury bill futures contract. For example, unlike certificates of deposit, commercial paper, bankers' acceptances or even Federal agency paper, each issue of Treasury bills is completely homogeneous. The only difference between 3-month Treasury bills issued in the first week of January, 1976, and 3-month bills issued in the third week of September, 1979, is their rates of interest. There is no variation in the quality of the credit behind the bills. They are the most standardized and homogeneous of all money market instruments.

Another characteristic which sets Treasury bills apart from other short-term securities is their high degree of liquidity. They are considered to be "near money" and, as a direct obligation of the U.S. Government, Treasury bills are among the lowest risk assets of all money market securities. Investors of all types, including commercial banks, pension funds, state and local governments, individuals, corporations and foreign central banks are familiar with Treasury bills as an investment vehicle and their low-risk, high-liquidity, interest-earning nature makes them an attractive financial security.

Due to their unique standing among all money market securities, Treasury bills serve as a general barometer of money market conditions. Treasury bill yields are highly correlated with the rates on other money market securities and changes in the bill yields are watched very closely as an indicator of changes which are likely to occur in the whole structure of short-term interest rates. Changes in Treasury bill rates receive a great deal of attention in the financial press.

The distinct characteristics of Treasury bills described above make them special in more ways than one. They are homogeneous in the same sense that hard red winter wheat or No. 2 yellow corn are homogeneous. They are highly liquid and familiar to virtually all money market participants and changes in their prices (yields) are usually indicative of the general movement of all money market instruments. These characteristics make Treasury bills the logical choice as the delivery vehicle of a futures contract based on short-term money.

The Treasury Bill Futures Market

Traded on organized exchanges, futures contracts are legally binding agreements which obligate the seller of the contract to deliver the commodity specified in the contract to the buyer at some future date at some fixed price. Each futures contract specifies a standardized quantity and quality of the commodity underlying the contract as well as the location and time of delivery on the contract. In other words, all physical aspects of the contract are standardized and homogeneous. The only negotiable variable on a futures contract is the price at which the contract is traded.

Two positions can be taken in a futures market. If a futures contract is purchased, the investor is said to be "long" in the futures market. By purchasing a futures contract, the holder of the contract is entitled to accept delivery of the standardized commodity specified by the contract at some future date at a price established when the contract was initiated. On the other hand, a futures contract may be sold, and in this case, the investor is said to be "short" in the futures market. When a short position is taken, the seller may deliver the specified commodity when the futures contract matures, at a price that was established when the contract was sold.

With the exception of price, the specifications of a futures contract are standardized and both the long and the short positions call for the acceptance or delivery of the same commodity bundle. Therefore, if an investor were to simultaneously buy and sell two identical contracts, his net obligations would be eliminated. However, if these two transactions were staggered over time, then although the physical obligations of the contract would offset one another, the net monetary difference between the two contracts may not. This points out the fact that the only negotiable variable on a futures contract is its price and the price of futures contracts change over time.

Specifications of the 90-day Treasury bill futures contract

The 90-day Treasury bill futures contract is similar to other futures contracts in that the terms of the contract are standardized with respect to the quantity and quality of the deliverable commodity and the location and time of contract maturity. Specifically, the 3-month (13-week) Treasury bill futures contract calls for the delivery or acceptance of a 3-month U.S. Treasury bill having a face value of $1,000,000 at maturity. Table 2.1 delineates the key specifications of the Treasury bill futures contract. At the seller's option, a delivery unit may be composed of U.S. Treasury bills bearing

Table 2.1. Specifications of the 90-day Treasury bill futures contract[a]

	ACE	COMEX	IMM
Delivery vehicle	$1 million par value of Treasury bills with 90, 91 or 92 days to maturity	$1 million par value of Treasury bills with 90, 91 or 92 days to maturity	$1 million par value of Treasury bills with 90, 91 or 92 days to maturity
Initial margin[b]	$800	$800	$1,500
Maintenance margin (per contract)	$600	$600	$1,200
Price Quotation	Complement of discount rate	Complement of discount rate	Complement of discount rate
Daily limits	50 basis points	60 basis points	50 basis points
Delivery months (each year)	January, April July, October	February, May August, November	March, June September, December
Date first traded	June 26, 1979	October 2, 1979	January 6, 1976

[a] Source: Arak and McCurdy, Federal Reserve Bank of New York, Quarterly Review (Winter 1979/1980).

[b] Margins vary according to whether the contracts cover speculative, hedged, or spread positions. The margins shown are speculative.

maturities of 91 or 92 days; however, all bills in a delivery unit must bear uniform maturity dates. For all practical purposes, the 13-week bill that is actually delivered on the futures contract is almost always the 91-day bill issued on Thursdays by the U.S. Treasury.

During each calendar year, there are four contract months on each of the exchanges that trade Treasury bill futures. On the IMM, delivery months occur in March, June, September, and December. ACE has delivery months in January, April, July, and October, and Comex in February, May, August, and November. Therefore, each month of the calendar year is a delivery month on one of the exchanges. On the IMM, each contract is currently traded for a period of two years; therefore, there are eight contracts trading at any one time. For example, in February, 1980, the following contracts were trading: March, 1980; June, 1980; September, 1980; December, 1980; March, 1981; June, 1981; September, 1981; and December, 1981. When the March, 1980 contract expired, a new contract for March, 1981, began trading. When all of the exchanges that trade Treasury futures contracts are considered together, an investor has the opportunity to buy or sell a contract for any calendar month extending almost two years into the future.

When a Treasury bill futures contract matures, delivery is made on the business day following the last day of trading. This is usually the third Thursday of the month. With respect to the IMM, delivery is made to a Chicago bank registered with the Exchange and a member of the Federal Reserve System. Payment for the securities delivered on the maturing contract is conducted through wire transfer of Federal funds.

The quotation of contract prices

The price of a futures contract is the major decision variable that must be considered when evaluating a trade in the futures market. The traditional method of quoting futures contracts results in a bid that is lower than the offer. However, for trading Treasury bills and other discount securities that are quoted on a yield basis, this relationship is reversed because of the inverse relationship between yields and prices. In order to make the method of quoting Treasury bill futures conform to traditional methods of trading commodities, the contract's price is quoted as the difference between 100 and the annual discount rate on the bill in question:

$$\text{IMM INDEX} = [100 - \text{Annual Yield}]\,. \tag{2.1}$$

For example, if a futures contract specifies delivery of 91-day Treasury bills to yield a 10.25 percent annual return, then the price of the contract will be quoted as 89.75 (100-10.25). This method of quoting the Treasury bill futures contracts preserves the traditional futures market relationship in which the long (short) position profits when the contract's price rises (falls). Since the index described above is based on an annualized rate, it is not the actual price that must be paid for the bill at delivery. The actual price paid is computed by using the annual rate of discount in the standard bill price formula as follows:

$$\text{Dollar Price} = \{\$100 - [\frac{\text{days to maturity}}{360} \times \text{Discount rate} \times \$100]\} . \quad (2.2)$$

In Equation 2.2, the discount rate is equal to the difference between 100 and the settlement price of the futures contract.

Structure of the Treasury bill futures market

The structure of a futures market may be divided into several categories; the exchanges and clearing houses, the futures commission merchants or commission houses, and the market participants or retail trade. Treasury bill futures are traded on the IMM in Chicago and on the ACE, and Comex in New York. Each of these centralized exchanges are headed by boards of directors elected by and from the membership and operated through a number of committees appointed by the directors. The exchanges are nonprofit organizations and their objectives are to: (1) establish equitable business conduct among members; (2) provide an organized market place and establish the time of trading; (3) provide uniform rules and standards for the conduct of trading; (4) establish uniformity of contract size and trade customs regarding quality and its establishment, time and place of delivery, and terms of payment; (5) collect and disseminate market information to members and the public; (6) provide a mechanism for the adjustment of disputes among members; and (7) provide machinery to guarantee the settlement of contracts and the payment of financial obligations in connections with trading among members.

Organized as a separate entity, the clearing corporations play a strategic role in reconciling all trades, managing the payment and receipt of funds and guaranteeing all futures contracts. By interposing itself between each buyer and seller, the clearing corporation assumes the opposite side of every transaction. When delivery on a maturing contract is made, the long position will pay the invoice total to the clearing corporation in return for the delivery vehicle. Similarly, the clearing corporation will remit to the short position the settlement value of the bills specified by the contract in exchange for the bills delivered on the contract. By this method, the proper settlement of each contract is carried out without it being necessary for each position to be individually matched to an opposite position. As a consequence, any sequence of transactions may occur; in other words, the investor may initiate a short position before a long position or visa versa.

In addition to the exchanges and the clearing corporations, the futures commissions merchants stand between the clearing house and the retail trade of the futures industry. They solicit and transact futures business with the public and carry out orders on the trading floor, or instruct others to do so if they are not, themselves, members of the clearing corporation. All futures commission merchants must register with the Commodity Futures Trading Commission (CFTC), the federal regulatory agency charged and empowered under the Commodity Futures Trading Commission Act of 1974 with regulation of futures trading in all commodities.

The final segment of the futures industry is composed of the retail trade which consists of numerous customers who buy and sell futures contracts in an effort to achieve their business goals. Many types of financial institutions participate in the Treasury bill futures market, however, private individuals not acting in a business capacity account for the major part of the open positions in this market. Table 2.2 shows the results of a survey by the CFTC of positions outstanding on March 30, 1979. The results show that "commercial traders," business interests other than the futures industry, accounted for only about one-third of the open interest in the Treasury bill futures market on the IMM. This is significant because it is the commercial traders more than any other group who would most likely use the markets for hedging purposes. On the other hand, two-thirds of the open interest is controlled by noncommercial traders, i.e., the futures industry, commodity pools and individual traders. This group includes those who speculate on interest rate levels and spreads and those who arbitrage differences between the two markets. These results suggest that much of the activity on the three-month Treasury bill futures market may be speculative and that the markets are not heavily used for their hedging potential.

Functions of a futures market

Futures markets emerge in response to the risks associated with price volatility of the cash commodity which underlies the futures market. The risk associated with holding an asset or security may be

Table 2.2. Participants in the 90-day Treasury bill futures market
Average open interest; number of contracts[a]

Occupation	1977 amount[b]	1977 as percentage of total[b]	1979 amount[b]	1979 as percentage of total[b]
Commercial traders				
Securities dealers	2,758	18.3	5,596	12.5
Commercial banks	326	2.2	1,581	3.5
Savings and loan associations	56	0.4	136	0.3
Mortgage bankers	44	0.3	974	2.2
Other	1,767	11.7	6,706	15.0
Total	4,950	32.8	14,992	33.6
Noncommercial traders				
Futures industry	2,765	18.3	8,434	18.9
Commodity pools	1,520	10.1	5,640	12.6
Individual traders	5,868	38.8	15,586	34.9
Total	10,154	67.2	29,661	66.4
TOTAL	15,104	100.0	44,654	100.0

[a]Source: Surveys November 30, 1977, March 30, 1979. The 1977 survey covered all positions, but the 1979 survey excluded positions of fewer than five contracts.

[b]Because of rounding, amounts and percentages may not add to totals.

divided into business risk and market (or systematic) risk. With respect to Treasury bills, business risk refers to the risk of default on the Treasury bills. This risk is minimal, however, because Treasury bills are backed by the U.S. Government. Market risk, on the other hand, is the risk associated with liquidating Treasury bills at a loss in the secondary market due to adverse price changes. The market risk of holding Treasury bills is positively related to the volatility of the yields on these securities. One of the major functions of a futures market is the division and transfer of risk. A futures market separates the market risk of a commodity from its business risk and facilitates the transfer of that market risk from those unwilling to accept it to others who desire it.

The reduction of market risk through the use of futures markets is carried out by hedgers in the market. A hedger uses futures contracts to manage the market risk exposure associated with the ownership of a cash commodity of the commitment to make a merchandising transaction in a commodity at some future date. This definition suggests that the hedger's principal business occurs in the cash market commodity and that his primary motivation for using futures contracts is to manage (reduce) the market risk of his cash commodity. Thus, he is presumably left with only the business risk to deal with, the evaluation and management of which is supposedly his comparative advantage.

A Treasury bill futures contract can be used to hedge the value of an existing portfolio of bills or a future purchase or sale of bills.

Whatever the specific strategy, the Treasury bill futures market can be used as a hedge against the risk associated with adverse interest rate movements. In order for hedgers to reduce market risk, there must be someone willing to accept it; hence, the importance of the speculator.

Unlike the hedger who matches a cash market position with a futures market position, the speculator takes a position in one market only. The speculator in futures markets fulfills several vital economic functions which facilitate trading in the underlying cash market security. For example, by risking his venture capital for the sake of profiting on an accurate forecast of a change in the futures price, the speculator provides the very important risk-shifting opportunity for the hedger. This suggests that the speculator's comparative advantage lies in predicting market price movements (which gives risk to market risk) rather than in the merchandising aspects of the underlying commodity (which leads to business risks). Besides providing for the transference of risk, the presence of many speculative buyers and sellers imparts a high degree of liquidity to the futures market that allows the hedger to buy and sell in large volume with ease. Finally, active speculation in futures markets tends to dampen the extremes of price movements that might otherwise occur.

Although the distinction between hedging and speculative activity in a futures market is not always well-defined, it is apparent that the risk-shifting functions of a futures market are made possible by the interaction between hedgers and speculators.

Trading futures contracts

To buy or sell a Treasury bill contract, a contract order is placed with a futures commission merchant who then sends it to the trading floor to be executed. For example, suppose that in December a commercial bank anticipates an inflow of deposits in six months time, June, and expects to invest those funds in 3-month Treasury bills maturing in September. Since the bank does not know what the 3-month Treasury bill rate will be in June when the bills are purchased, it can hedge its future merchandising commitment by going long in the June 90-day Treasury bill futures contract. This is illustrated in Table 2.3.

Table 2.3. Long hedge in 90-day Treasury futures

Date	Cash Market	Futures Market
December	Bank plans to purchase $10,000,000 of 3-month T-bills in June Current price of 3-month T-bills: $98.00	Bank buys 10 June contracts IMM index: 91.50
June	Bank purchases $10,000,000 of 3-month T-bills maturing in September Price paid: $98.25	Bank sells 10 June contracts IMM index: 92.50
	Loss: 25 basis points	Gain: 100 basis points

In the above illustration, the bank had to pay a price 25 basis points higher on the bills it bought in June than the December price of bills. Thus, the bank had an additional expense of $25,000. However, since the bank hedged its future purchase of Treasury bills by purchasing ten June futures contracts, it was able to offset the loss incurred in the cash market with an equal gain in the futures market. This exposition makes the extreme assumption that the futures index moved in a ratio of four to one relative to the cash price, and in the same direction. Had the relative price changes been something other than four to one, or in opposite directions, then the gains in the futures market would not have been identical to the loss in the cash market. This points out the fact that it is the relative price relationship between spot and futures that is important in hedging, not the absolute level of prices.

In a similar situation, had the bank anticipated selling bills out of its portfolio rather than purchasing them, then the bank could have hedged against a market decline in bill prices by going short in the futures market. In either case, the bank would be attempting to reduce the market risk associated with adverse price movements by hedging its anticipated cash transactions in the futures market.

The hedging example described above illustrates several important features of futures markets transactions. First of all, the position taken in the futures market reverses the sequence of a normal cash transaction. If a bank holds an inventory of cash bills which it plans to sell in three months, then in the meantime, the bank is said to be long in cash securities. To hedge its inventory, however, the bank goes short in the Treasury bill futures market. If cash and futures prices move in a parallel direction, then the fact that the bank took an opposite position in the futures market to its long cash position means that a loss in one market will be at least partially offset by a gain in the other market.

The hedging example illustrated above showed that in June, the bank purchased Treasury bills in the cash market and sold contracts to offset its futures market obligations. Unlike real assets that may be held indefinitely, bill futures contracts expire within a given amount of time. The ultimate disposition of a futures contract may occur in one of two ways: by offset or by delivery. In the case of offset, a long (short) futures position is liquidated by selling (buying) an equal number of identical contracts to the initial position. This makes the net futures position equal to zero, cancelling all contract obligations. On the other hand, contract obligations may be satisfied at the contract's maturity date by accepting delivery of bills (on a long position), or delivering bills (on a short position). Delivery procedures are fully specified by the various exchanges. For example, on the IMM, delivery occurs four times each year when the March, June, September and December contracts mature. The contracts cease trading on the third Wednesday of each contract month and delivery occurs on the following Thursday. When bills are delivered on a maturing contract, the price paid for them is based on the settlement price of futures contract on its last day of trading.

Profits or losses on a Treasury bill futures contract occur similarly to any other transaction. The gain or loss is simply the dollar equivalence of the change on the price of the contract over time. For example, if a contract were purchased at 93.00 and liquidated six months later at 93.85, the dollar gain would be based on a price rise equal to .85. It is not necessary, however, to purchase a futures contract before selling it. Profits or losses can also be made on a short futures position where the futures contract is sold first and repurchased at a later date. In this case the gain (loss) is calculated on the difference between the initial price at which the contract was sold, and the ending settlement price at which the contract was repurchased.

Costs of futures trading

The costs of futures trading may be divided into two categories: commissions and margins. Commissions on futures contracts are assessed on a "round turn" basis--that is, the commission covers both entry and exit from the futures market. The commission is paid by the customer to the futures commission merchants when the futures position is closed out. All positions, long or short, must pay the commissions charges. From the inception of the IMM's 90-day Treasury bill futures contract on January 6, 1976, to March 4, 1978, the date when negotiated commissions became effective, the minimum commissions structure for the contract remained constant at $39.00 for "day trades" and $60.00 for "overnight" positions. Although commission rates are currently negotiable, they are not substantially less than the former nonnegotiable rate unless the volume of contracts traded is large.

In addition to the commission charge, margin money must be posted. The "initial margin" serves as a security deposit or earnest money guaranteeing the performance on the contract and must be posted by both long and short positions at the initiation of their contracts. Members of the clearing house must post a margin of $1,200 per contract, which can be in the form of cash or bank letter or credit. The clearing member firm must, in turn, impose an initial margin requirement of at least $1,500 on its customers. This may be posted in the form of cash, selected Treasury securities, or in some cases, letters of credit. The initial margin, which is posted when the futures contract is opened, may be interpreted as the investor's equity position in the futures market. If the investor is long (short) in futures contracts and the price of those contracts rises (falls), then the equity position is enhanced and profits in the margin account may be withdrawn immediately. On the other hand, in the event of adverse price movements, i.e., a fall (rise) in contract prices, the margin account will be impaired. When losses occur and reduce the firm's margin below a specified level call the "maintenance margin," the firm must replenish its margin account to its original level.

Maintenance margin is usually set at 75 percent of the initial margin; in the Treasury bill futures market, the maintenance margin is currently $1,200 per contract, at a minimum. The purpose of specifying a maintenance margin is to ensure the financial integrity of the clearing house by protecting the contract holder against large accumulated losses. If the investor is long (short) in futures and the price of his contracts falls (rises) to such an extent that the value of his margin balance decreases to less than the maintenance margin level of $1,200, he will be advised to restore his margin account back to the original level of $1,500, the initial margin. These "margin calls" must be made in cash to the brokerage firm before the commencement of trading on the next business day.

The daily resettlement procedure of crediting or debiting the margin balance according to the daily price movements of the futures contract is called "marking-to-market." For as long as the futures position is outstanding, the contract will be marked-to-market by the clearing house at the end of each business day.

To illustrate the mechanics of the margin account, suppose that a bank plans to purchase $1,000,000 worth of 3-month Treasury bills in September, six months from now. Suppose also that the current price of the futures contract maturing in September implies a yield on its delivery vehicle that the bank finds quite satisfactory. Therefore, the bank plans to purchase one September Treasury bill futures contract and accept delivery on the contract. This is illustrated in Tables 2.4 and 2.5, along with an exposition of the margin account. For the sake of simplicity, it will be assumed that the

Table 2.4. Futures market long hedge and exposition of margin account when contract price rises

Margin Account	Futures Transactions
March 1	March 1
Deposit $1,500 initial margin with futures commission merchant.	Buy one December 90-day Treasury bill futures contract.
	Contract price: 92.00
June 15	June 15
$1,500 initial margin 250 margin profits withdrawn	Contract's price rises to 92.10
$1,500 total margin	
September 23 (delivery date)	September 23 (delivery date)
$1,500 initial margin 500 margin profits withdrawn	Final contract settlement price: 92.20
$1,500 total margin	

maintenance margin is set at $1,500, the same level as the initial margin. Therefore, each basis point change in the price of the contract will trigger either a margin call, if the price declines, or the withdrawal of margin profits, if the futures price moves higher.[1]

In the example illustrated in Table 2.4, the bank goes long in the

[1]On a Treasury bill futures contract worth $1,000,000 of face value, each basis point change in the price of the contract is worth $25.00 (.0001 (90/360) 1,000,000). Thus, if the initial margin is $1,500 and the maintenance margin is $1,500 as well, then for each basis point decline (rise) on a long (short) futures position, the contract holder will be required to post a margin call of $25.

futures market on March 1 by depositing $1,500 in initial margin with its futures commission merchant. By June 15, it is shown that the contract price has risen by 10 basis points yielding a margin account profit of $250. Continuing to rise until the time of delivery, the final settlement price of the contract is 92.20, a 20 basis point rise since the original purchase price of the contract. Thus, with respect to the margin balance, the bank has made a total profit of $500 over six months.

Accepting delivery of cash bills on the contract, the bank pays to the clearing house an amount corresponding to the final settlement price of the futures contract, 92.20. This being 20 basis points above the price of the contract when it was purchased, the bank incurs an added cost of $500 in the price it must pay for the cash bills. However, since the margin account shows a $500 profit over the six month holding period, the effective price on the cash bills is still 92.00. Having taken delivery on its contract, the bank has fulfilled its contract obligations and therefore, the initial margin (performance bond) that was paid to the brokerage firm is returned to the bank.

The analysis of Table 2.5 is similar to the above except in this instance, the contract's price declined, the margin balance was impaired and the bank was required to make margin calls to replenish the value of its account. However, in this case, the $500 loss sustained in the margin account was offset by a contract settlement price that was lower than contract price at which the bank entered the

Table 2.5. Futures market long hedge and exposition of margin account when contract price falls

Margin Account	Futures Transactions
March 1	March 1
Deposit $1,500 initial margin with futures commission merchant	Buy one December 90-day Treasury bill futures contract Contract price: 92.00
June 15	June 15
$1,250 initial margin 250 margin calls $1,500 total margin	Contract's price falls to 91.90
September 23 (delivery date)	September 23 (delivery date)
$1,000 initial margin 500 margin calls $1,500 total margin	Final contract settlement price: 91.80

futures market. Again, as in the first case, the effective price is 92.00 and again, since all contract obligations were met by accepting delivery, the initial margin money is returned to the bank.

The Forward Market

Forward transactions are common in many areas of economic activity, including the money market. In a forward transaction a seller agrees to deliver goods to a buyer at some future date at some fixed price. For example, a bank may forward contract to sell Federal funds a few days hence based on a price specified today.

The forward contract is a tailor-made agreement designed to suit the individual needs of the agreeing parties. Specified in the contract are the terms of the agreement which include the price, quantity, quality, time and place of delivery, and the terms of payment. Since each forward contract is unique with respect to the terms listed above, the forward contract market is thin and heterogeneous. Thus, forward contracts are not traded on an exchange but in an informal, decentralized market. For this reason, forward contracts are not guaranteed in any way except by the faith and credit of the agreeing parties.

Futures vs. forward

Forward contracts for deferred performance and exchange of title have characteristics which are both similar and dissimilar to futures contracts. Like the futures contract, the forward contract specifies the price, quality, time of delivery, etc. of the underlying commodity, however, this is where the similarities end.

Unlike forward contracts that are heterogeneous in character, futures contracts are standardized agreements which are identical and homogeneous with respect to all terms except the month of delivery and the price. For this reason, futures contracts are traded on formalized exchanges governed by detailed rules which are enforced by a professional staff of the exchange. Since futures contracts have the backing of the clearing house, the performance on the contracts is guaranteed by the exchange. Forward contracts, on the other hand, have no guarantee other than the pledge made by the agreeing parties. As a consequence, the futures market has more depth and liquidity than the forward market.

Another consequence of futures trading on an organized and centralized exchange is the fact that, unlike the forward contract, the futures contract is a negotiable instrument. To liquidate an open contract in the futures market simply involves taking an offsetting position; however, in the forward market, this is not generally possible since forward contracts are tailor-made, non-negotiable agreements.

This suggests another distinction between forward and futures contracts. Forward contracts are usually made with the full intention to accept delivery on the contract, with title being transferred. Futures contracts, on the other hand, are usually offset prior to their maturity with no title ever being exchanged.

A final distinction between futures and forward contracts that is significant for later analysis is the way in which profits or losses are made. In the futures market, profits or losses are accumulated in the margin account over the holding period of the open contract. This daily resettlement process is not present in the forward market since the forward market requires no margin account. All profits or losses in the forward market are incurred at the time delivery is made on the forward contract.

Thus, with respect to profits or losses, a major distinction between the futures and the forward markets is the timing over which they are realized.

Market Efficiency and Arbitrage

Two concepts that will be pervasive throughout the analysis are the concepts of market efficiency and arbitrage. For this reason, they are introduced in the present context because they apply to both the futures market and the cash market.

Market efficiency

An efficient market is one in which current market prices reflect all available and relevant information. As new information becomes available, it is immediately incorporated into the market resulting in changes in market prices. For example, suppose that new information acquired by traders causes them to anticipate that the futures price of a contract, FP_t, will rise in the next period, t+1, to FP_{t+1}. They will then be willing to purchase that contract as long as the present discounted value is greater than the current price of the contract, i.e., $FP_t < FP_{t+1}/(1+r)$, where r is the appropriate rate of discount. If the above inequality holds, traders would purchase the contract at FP_t and sell it at a later date for a price equal to FP_{t+1}. In the aggregate, competition would ensure that the current futures price would be bid up to a point where no profit would be made by purchasing the contract in the current period and selling it in a future period, that is, to the point where $FP_t = FP_{t+1}/(1+r)$. Similarly, if the present discounted value of the futures contract in period t+1 were less than the current price of the futures contract, $FP_t > FP_{t+1}/(1+r)$, traders would sell the futures contract until the current price of the contract and the present discounted value were equal, $FP_t = FP_{t+1}/(1+r)$.

The efficient market hypothesis states that all currently available and relevant information about current and future events is reflected in current market prices. Thus, as new information about future events becomes available, expectations of future prices will change, causing changes in current prices. If new information comes to the market in the form of a random series of events, and if market prices change quickly according to revised expectations, then it follows that the resulting time series of prices will themselves exhibit a random process. Thus, a time series of price changes in an efficient market should exhibit a random walk and conform to the assumptions of the classical linear model concerning the error variance. Specifically, econometric results of a regression of current prices on previous period prices should show that in an efficient market, the vector of residuals (the first differences in prices) should be independently distributed with no evidence of serial correlation. Accordingly, one major proposition of the efficient market hypothesis states that the best estimate of next period's price is the current period's price.

Algebraically, the efficient market hypothesis may be expressed as the following Martingale sequence.[1]

$$E(P_{t+1}|\phi_t) = P_t[1 + E(r_{t+1}|\phi_t)] + E(e_{t+1})$$

where:

P_{t+1} = the price in period t+1,

P_t = the price in period t,

r_{t+1} = the discount rate in period t+1,

ϕ = the information set available at time t,

e_{t+1} = the residual error term in period t+1, and

E = the expectations operator.

By lagging all of the variables by one period, the efficient market hypothesis may alternatively be represented by the second-order Martingale sequence:

$$E(P_t/\phi_{t-1}) = P_{t-1}[1 + E(r_t/\phi_{t-1})] + E(e_t)$$

where, in accordance with the assumptions of the classical linear model, $E(e_t) = 0$, the variance-covariance matrix, $E(e_t e_{t,t-j}) = I\sigma^2$ and I is an nxn identity matrix and $j \neq 0$.

One implication of the efficient market hypothesis is that a time series of price changes will approximate a random series. Therefore, much of the empirical analysis of this hypothesis has been concerned with testing for serial covariances of the observed price series (or returns). These tests have been based on various assumptions concerning the information set Φ_t. Weak form tests assume that

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[1]For a good summary of the efficient market model, see Eugene F. Fama (1976).

the information set consists of past prices (or returns) only, semi-strong form tests concern the speed of price adjustment to currently publicly available information (announcements of annual reports, monetary policy, etc.) and finally, strong form tests are concerned with the possibility that certain investor groups have monopolistic information relevant to price formation and the effect of this insider information on the determination of prices.

In this study, the concept of efficiency is applied to the interrelationship between the cash and futures market for 90-day Treasury bills. Efficiency, in the present case, implies that prices in the cash and futures market are structured in such a way that arbitrage between the two markets is not possible. One of the implications of efficiency under these circumstances is that the two markets will exhibit a direct and well-structured price relationship with the result that any aberration occurring in one market would be transmitted to the other market. Whether this in fact occurs depends on the degree of market efficiency, or alternatively, the existence or not of arbitrage.

Arbitrage

The concept of efficiency is defined, for purposes of this paper, in terms of the absence of arbitrage opportunities. Strictly defined, arbitrage means buying something where it is cheap and selling it where it is dear. For example, a bank may purchase money in the C-D market and sell it in the Eurodollar market and earn a profit equal to the net difference between the lending and borrowing rates. The profit is calculated ex ante with certainty and locked in by the arbitrage operation.

Arbitrage between the Treasury bill cash and futures market may take the form of either "pure arbitrage" or "quasi arbitrage." Pure arbitrage refers to shorting securities not owned and using the proceeds to fund an equivalent position in other securities at a lower price. In terms of the Treasury bill cash and futures market, this essentially means shorting Treasury bills in the cash market to fund an equivalent position using cash market securities and futures contracts. In this case, arbitrage profits are made without having to own the securities outright. Quasi arbitrage, on the other hand, refers to selling securities from an existing portfolio in order to finance an economically equivalent position composed of both Treasury cash securities and Treasury futures contracts.

Although both forms of arbitrage exhibit the basic concept of buying cheap and selling dear, the transactions costs incurred in each type of arbitrage are not the same. In the case of pure arbitrage, the securities that are shorted must somehow be financed. Whether they are borrowed or obtained on a reverse repurchase agreement, the costs of financing the short position must be considered in any ex ante calculation of arbitrage profits. In the case of quasi arbitrage, explicit carrying costs do not exist because the securities that are sold already exist in the portfolio. However, as with pure arbitrage, there are additional transactions costs associated with the execution of the futures and cash positions. These include commissions and margins in the futures market and the bid-ask spread in the spot market.

The exposition of the arbitrage criterion of efficiency is a direct result of the development of the theory of the equilibrium price of a futures contract. If the structure of prices in the futures and spot market for Treasury bills is in equilibrium, neither pure arbitrage nor quasi arbitrage opportunities will be available between the two markets. If, on the other hand, the price structure is not in equilibrium, then continuous attempts to engage in arbitrage will stimulate a dynamic process which will encourage market efficiency.

CHAPTER III. REVIEW OF LITERATURE

Futures trading in Treasury securities represents an innovative application of the traditional concepts of hedging and speculation. These concepts, as well as the benefits and costs associated with futures trading, have been thoroughly studied for the agricultural commodities over a long period of time by a wide variety of authors; Boyle (1920), Hoffman (1932), Blau (1944-45), Telser (1955-56), Gray (1960), Hieronymus (1971), Working (1948, 1949, 1953a,b, 1962 and 1970), Johnson (1957, 1960), Peck (1975), Stein (1961) and Ward (1971). Such is not the case, however, with the Treasury futures markets which are very new and still unfamiliar to many people, including potential users of the markets. Very few in-depth studies have been conducted, and the "hows" and the "whys" of market use as well as the costs and benefits of trading Treasury futures are still in the exploration stage.

General Analysis

The one notable exception to the otherwise dearth of comprehensive studies is a report conducted by the U.S. Treasury and the Federal Reserve System submitted to the Commodity Futures Trading Commission on May 14, 1979. The Treasury/Federal Reserve Study of Treasury Futures Markets (1979a,b) consists of two volumes which attempt to identify the major questions of social concern regarding the Treasury futures markets. Its analytical approach to the questions posed ultimately leads to a summary and conclusion regarding the potential benefits and problems of trading in Treasury futures. The study concerns itself with the 90-day Treasury bill futures as well as the one-year bill futures Treasury note and Treasury bond futures contracts. To date, the Treasury/Federal Reserve study is the only comprehensive analysis of the Treasury futures markets which attempts to analyze the broad social questions that arise with respect to trading in Treasury securities. For this reason, the study is examined for its value in delineating the general outline of the Treasury futures markets, and for exposing the major issues of social concern within its periphery.

The study was conducted in order to advise the CFTC of the Treasury's and the Federal Reserve's positions and recommendations regarding existing contracts and proposals for new ones. The report begins by outlining the major questions that the Treasury and Federal Reserve feel are the issues of social concern. The first issue concerns the effect that futures trading in Treasury contracts will have on the underlying cash market for government securities. Before the Treasury and Federal Reserve can endorse the initiation of new contracts, they want to be sure that futures trading in U.S. Government securities does not adversely affect the efficiency or integrity of the underlying cash market for these securities.

Specifically, the two agencies are concerned that speculation in the futures market may be destabilizing with the result that arbitrage between the two markets will transmit these destabilizing influences to the cash market. Their fear rests on two basic assumptions, neither of which, however, is fully analyzed in the study. The first is that speculation has a destabilizing effect on prices rather than a stabilizing one, and the second is that there is a structural link between prices in the two markets that exists due to arbitrage between the markets. Although the study does recognize the possibility that the first assumption may not be valid, it does not examine the nature of the price structure between the two markets. This, however, is essential in order to assess the effects that changing futures prices have on the cash market.

The second issue raised in the report is related to the first except that it concerns the effect that futures trading has on the Treasury's ability to manage the Nation's debt. Specifically, the study asks whether trading in futures contracts which depend on deliverable supplies of Government securities is likely to constrain the Treasury in its debt management decisions. With respect to the 90-day Treasury bill contract, the Treasury is worried that because there is only one issue deliverable on the maturing contract, it is possible that at the time of delivery, the total amount of bills specified by open long contracts may exceed the total supply of bills deliverable on the contract and that in this event, the cash market for 90-day bills would be seriously disrupted. This situation may occur as the result of market participants cornering the market. Although the Treasury recognizes that it could avert disruptive influences on the cash market by increasing the supply of deliverable bills at its weekly auction, the Treasury's primary goal is the efficient management of the Nation's debt, and it should not feel constrained to satisfy the circumstantial needs of participants in the futures market.

Given that futures trading has the potential for causing problems in the cash market for government securities, the third issue raised by the Treasury/Federal Reserve study is that of surveillance. The report questions the ability of the CFTC to maintain effective surveillance of the financial futures markets, particularly given difficult delivery situations and also as essentially duplicative contracts are traded simultaneously on several exchanges. The body of rules on most exchanges usually include emergency powers which can be enacted in times of trading disruptions. Furthermore, the CFTC is empowered by the Commodity Exchange Act of 1974 to ensure that the exchanges enforce their own rules, and direct an exchange to take any action needed to maintain orderly markets whenever it believes an "emergency" such as market manipulation exists. The Treasury/Federal Reserve report is concerned, however, that the CFTC may not be as diligent or as strong as it should be to oversee these markets.

The final issue raised by the report concerns the danger that unsophisticated investors will not fully appreciate the risks inherent in futures contracts whose names (erroneously) suggest the backing of the U.S. Treasury. Participation in the Treasury futures market involves highly leveraged positions of futures contracts and large profits or losses can be made very quickly. Although the contracts are guaranteed by the exchanges on which they trade, they do not in any way have the backing of the U.S. Treasury. The Treasury and Federal Reserve System are concerned with the adverse effect on public welfare that may result when unwary investors enter the futures market without a full appreciation of the risk and dangers involved in trading futures contracts.

Having outlined the above issues, the Treasury/Federal Reserve report goes on to explicate the potential benefits and problems resulting from trading futures contracts in Treasury securities. According to the report, the two primary benefits are: 1) the potential for the reallocation of risk from financial institutions desiring to avoid it to those willing to accept it, and 2) the aggregation and dissemination of information regarding the market's expectation of what future interest rates will be. This information is disseminated through the price system of futures contracts.

Potential problems arising from futures trading in Treasury securities include the same kind of issues that were outlined above. The destabilizing effect that futures speculation may have on the spot market, the potential disruption from market corners or squeezes and the question of adequate supplies of deliverable securities are among the potential problems which, according to the Treasury and Federal Reserve, may have the effect of lowering the public's welfare.

In addition to outlining the potential benefits and problems from futures trading in Treasury securities, the Treasury/Federal Reserve study also lists various conclusions and recommendations to the CFTC. With respect to the problem of the adequacy of deliverable supplies, the Treasury and Federal Reserve advocate a "market basket" approach to the question of the appropriate contract delivery vehicle. Rather than having a single Treasury issue be the sole delivery instrument, as is the case in the 90-day Treasury bill futures contract, the Treasury advocates the use of a contract which specifies delivery of any one of several Treasury issues (i.e., from a market basket), as with the Treasury bond contracts. This would effectively increase the deliverable supply of the contract security, thereby preventing problems due to inadequate supplies of deliverable issues. As a consequence, this would, in turn, reduce a potential source of disruptive effects on the cash market. With respect to the 90-day bill contract, the Treasury/Federal Reserve study of Treasury Futures Markets states that (p. 23):

> Because the 3-month bill contract has become so well established and so actively used in its present form, a redefinition of deliverable supply at this juncture seems unwarranted . . . . However, in view of the concerns expressed by market participants that the 3-month contract has been vulnerable to squeezes under certain conditions, steps should be taken to minimize these possibilities through improved data collection and monitoring of interactions between the futures and cash markets.

A second recommendation made to the CFTC concerns the potential risks associated with contract proliferation. The Treasury and Federal Reserve System have urged the CFTC to proceed slowly in authorizing new contracts. On specific issues, the CFTC has been urged to approve only those contracts which specify a market basket of delivery instruments and contracts which are not duplicative of ones already in existence.

Finally, the report recommends that safeguards for the protection of investors be instituted within the futures industry. These would include procedures for the surveillance of small dealer firms and the dissemination of information to alert the public of the risks of having a highly levered position. In addition to these, the report recommends that the exchanges establish customer suitability standards and other measures such as higher margin requirements and position limits to ensure investor protection.

The report issue by the Treasury and Federal Reserve System is intended to be a comprehensive summary of the major issues regarding trading in Treasury futures. It defines the boundaries of the field and the nature of the game, but not all of the intricacies of the rules. Many of these are assumed away. For example, it is apparent that the Treasury is alarmed by the potential effect that aberrations in the futures market (destabilizing speculation, market squeezes) may have on the spot market. And, it assumes that futures markets effects are transferred to the spot market via arbitrage. However, the report does not investigate the question of arbitrage or the price structure that exists between the futures and the cash markets. If a well-structured price relationship does not exist between the two markets, then it is not entirely clear that fluctuations in one market will be transmitted to the other market, as is feared by the Treasury. If, on the other hand, it can be shown that a definite price linkage does exist, then the Treasury's fears have a more solid foundation. In any case, it is essential to analyze and evaluate the formal structural price relationship between the futures and the cash markets.

Specific Studies

The futures-cash price relationship has been studied recently by a number of economists, each of whom has attempted to empirically evaluate the futures market using theories of the term structure and other hypotheses for determining the equilibrium value of a futures contract.

Poole Study

Poole (1978) analyzed the link between the futures and cash markets by constructing an arbitrage model using the nearby futures contract for 90-day Treasury bills and the 3- and 6-month cash bills. The model develops the condition for the equilibrium futures price by proposing an arbitrage situation between the futures market and the cash market where cash prices are treated as predetermined. The underlying hypothesis of the model is that yields on alternative investments which have identical holding periods should be the same. Ignoring transactions costs, Poole contends that a strategy consisting of cash market securities only will give rise to a forward rate which will be identical to the rate on a futures contract. Expanding the model to include the fixed costs of commissions and initial margin, he then develops an equilibrium range of futures prices between which arbitrage is not possible.

His empirical analysis is conducted by plotting the theoretical upper and lower critical yields of this arbitrage range on a daily basis for nearby contracts for the period from January 6, 1976, to June 23, 1977. Superimposed over the range of theoretical values are the observed yields on futures contracts. Poole concludes that although there is a tendency for the observed futures rates to fall closer to the lower than the upper arbitrage points, as well as a tendency for all rates to fall in the final contract month, it is apparent that profitable arbitrage opportunities rarely exist for the nearby contract and are small when they do exist. These results imply that during the three months prior to the contract's maturity, the only period of time when perfect arbitrage may be obtained, the futures contract is priced efficiently.

Poole's study was one of the first to analyze the equilibrium futures price. The study was limited, however, in its scope. Only the nearby contract was analyzed and various costs associated with the margin account were not included. Thus, the conclusions based on his study should not be regarded as general conclusions pertaining to all contracts over their entire trading period.

Emery and Scott Study

In an analysis which is similar to Poole's, Emery and Scott (1979) attempt to provide an indirect test of the pure expectations hypothesis of the term structure of interest rates. In their analysis, they implicitly assume that a forward contract is identical to a futures contract. Thus, they set out to prove that if the yields on futures contracts are similar to those on the corresponding forward contracts, then it can be concluded that the value of the forward rate as determined by the pure expectations hypothesis is, indeed, an unbiased estimate of expected rates, that is, the futures rates. Of course, they are assuming that the futures rates are unbiased expectations of futures spot rates. This may not be true; in any case, it is an empirical question.

Using weekly spot rates to calculate forward rates and ignoring all transaction costs, Emery and Scott plotted a time series of forward rates against observed futures rates. Although they applied no formal test of the significance of the difference between the two measures of the expected yields, they nevertheless conclude that "the two types of estimates seem to lie well within a range such as to encourage rejection of the null hypothesis that a significant difference exists." Their eye-ball observations apparently give them enough confidence to conclude that the rates observed on the futures contracts are merely a second observation of the same market expectations implied by the current term structure of interest rates. In fact, they go even further to conclude that even for securities that do not have futures markets, the rates implied in these securities would probably be very similar to those that would come to exist if futures trading were established in that market.

Emery and Scott's study is less than rigorous. Their assumption that futures rates are unbiased expectations of future spot rates is not supported in their work and their conclusions have no foundation upon statistical testing procedures. Therefore, their off-hand conclusion that observed futures rates substantiate the expectations hypothesis of the term structure must be regarded with caution.

Lang and Rasche Study

Lang and Rasche (1978) also provide a similar analysis to that of Poole's; however, they arrive at very different conclusions. The conceptual approach is essentially the same as Poole's in that they postulate that the yields on 3-month bill futures are identical to the forward rates for comparable periods, but their empirical analysis is far more extensive. Rather than constraining themselves to an analysis of the nearby contract only, they compare the forward and futures rates over the full life of contracts extending two years into the future. Since Treasury bills have a maturity of one year or less, they are forced to use coupon securities to calculate forward rates comparable to all futures contracts that mature in one year or more. In addition, they encounter the problem that the maturities on many of the cash securities do not properly match the relevant dates on the futures contracts. Thus, they are forced to approximate their calculations, by interpolating (mismatched) forward rates in order to arrive at estimated forward rates that are exactly comparable to the rates on futures contracts.

The comparison of the forward and futures rates is conducted on a bond equivalent basis for three separate periods from March 1, 1976, to March 31, 1978. For each period, the absolute and arithmetic mean of the difference between the futures and forward rates are calculated for each of eight successive contracts, from the nearby to the most distant. The analysis is conducted across contracts and over time in order to determine whether contract maturity or time trends have any effect on the relationship between the futures and forward rates. Based on their tests of the significance of the absolute difference between the futures and the forward rates, Lang and Rasche come to the conclusion that the forward and futures rates are not equal, contrary to Poole's conclusion. By analyzing the arithmetic difference over time, they further conclude that the difference between the two rates has not narrowed over time as one might suspect they would as the market matures. In particular, the two researchers find that for the first two contracts nearest to delivery, the observed points that fall outside of the no-arbitrage range usually fall below it whereas for the later-dated contracts, the observed rates outside of the arbitrage range almost always are above it.

Lang and Rasche refute the findings of Poole and find that for contracts near to their maturity, the futures rate is biased downward relative to the forward rate and that this situation is reversed as the time to the contract's maturity is increased. In attempting to explain this bias, they point out that the default risk of an implied forward contract (which is constructed using U.S. Treasury bills) is different from that on a futures contract. Whereas the Government's backing of U.S. Treasury bills makes them default free, futures contracts which are only guaranteed by the exchanges on which they trade are not. Therefore, they suggest that investors in futures contracts may require a risk premium for the more distant contracts with the consequence that the futures rate will be biased upward relative to the forward rate.

Although Lang and Rasche extended Poole's analysis by analyzing the entire (two year) trading life of each contract considered, they continued to neglect certain differences between futures contracts and forward contracts which may be significant for the analysis of the futures and forward rates. Futures exchanges require daily resettlement of margin accounts; forward transactions do not. This fact introduces an element of uncertainty into futures trading which is not captured in the above analysis.

Puglisi Study

In a study analyzing investment strategies, Puglisi (1978) examines the case of quasi arbitrage and compares the holding period returns of alternative strategies. The first involves purchasing a bill and holding it to maturity and the second consists of the tandem position of a second cash bill (91 days greater maturity than the first bill) and a short futures position which has a common maturity date with the first cash bill.

Puglisi ignores all transactions costs in his empirical analysis which uses daily settlement prices on nearby futures contracts and (dealer-ask) prices on cash Treasury bills. Using t-statistics for testing for the significance of the difference between returns on the bills-only strategy versus bills-futures strategy, Puglisi concludes that the difference is significantly greater than zero. Based on his empirical results, he concludes that the futures market for 90-day Treasury bills is inefficient. Trying to explain the bias between the two rates of return, Puglisi suggests that institutional constraints which prevent entry of institutions which could arbitrage the market may be one reason that the market inefficiencies continue to exist. This further suggests, he contends, that institutional investors who are free to use the markets can increase the returns on their portfolios by developing strategies that involve bills-futures as well as bills-only.

Puglisi's approach suffers the common error of ignoring certain variables that may be significant for the analysis. These include the fixed costs of commissions and initial margin and the variable costs that are associated with the margin account.

Vignola and Dale Study

In a comment to Puglisi's analysis, Vignola and Dale (1979a) take issue with the manner in which his results are reported. Vignola and Dale calculate the same rates of returns for the strategies as proposed by Puglisi; however, they extend the analysis to the first three contract maturities rather than just the nearby contract. They find that although their numerical results differ from those obtained by Puglisi, they concur with his conclusion that the futures market is inefficient. Their major concern, however, is in the method of testing and reporting the results. Puglisi reported summary statistics which included, among other things, the mean difference between the returns on alternative strategies for various futures contracts and the standard deviation of those returns. Vignola and Dale contend, however, that the distribution of the difference in returns from bills-only and bills-futures strategies must be examined on a daily basis in the form of a daily time series. To this end, they provide diagrams of daily time series of the bills-futures returns minus bills-only returns for each contract. The diagrams showed that summary statistics which average the difference can be (and are) misleading especially if the difference in returns reverses its sign over the period of analysis. The results also indicate that there is significant auto-correlation in the arbitrage returns for each contract, confirming that the futures market is inefficient, not only in an arbitrage sense but also in the sense that arbitrage returns are not distributed randomly over time. Although their major conclusions are similar to Puglisi's, Vignola and Dale point out that investors buy and sell on individual days, not at the mean return and thus, it is the daily time series of alternative returns that is important for the portfolio manager, not summary statistics.

Vignola and Dale's point that investment returns should be analyzed on a daily basis rather than in summary form is well taken.

This is apparent from observation of the empirical results. However, their study suffers similar shortcomings as Puglisi's due to the fact that the two methodologies are essentially the same.

Capozza and Cornell Study

Capozza and Cornell (1979) examine the case of pure and quasi arbitrage and set out to test the familiar hypothesis that the futures rate on a contract should be equal to the forward rate corresponding to that contract. Like Puglisi, and Vignola and Dale, Copozza and Cornell exclude transactions cost in their model in any explicit way, although they do qualitatively recognize their impact on the differential between futures and forward rates. To simplify the exposition of their arbitrage condition, continuously compounded rates of return are used rather than the Treasury bill discounts normally quoted in the financial press. Thus, their forward rates are derived by taking the natural log of the prices of adjacent Treasury bills and the futures quotes are converted to continuously compounded rates as well.

Tests of the arbitrage condition were based on the differences between the futures and forward rates using weekly data. The data were limited to the first three futures contracts, since beyond that horizon, it would have been necessary to use coupon securities to calculate forward rates with the result that they would not have been directly comparable to the futures rates. Summary statistics included the average deviation and the average absolute deviation between the futures rate and the forward rate. The averages were computed by taking the first three futures contracts, subtracting the relevant forward rate from the futures rate, and averaging the difference over the thirteen weeks in each quarter of the sample period, March, 1976, to June, 1978. Also reported in the results are tables and graphs which show the relationship between the deviation and the maturity of the futures contract. The results clearly show that the futures rate is biased downward relative to the forward rate for the nearby contract, but as the time to maturity of the contract increases, the futures rate becomes biased upwards relative to the forward rate. Capozza and Cornell attribute the existence of this differential to two institutional constraints: (1) the costs associated with shorting securities in the cash market, and (2) the reluctance of institutions to enter the futures market.

Like the previous authors, Capozza and Cornell fail to incorporate in their analysis all of the variables that are likely to influence the futures rate relative to the forward rate. They ignore the fact that a futures contract is not economically equivalent to a forward contract and the implication of this difference.

Rendleman and Carabini Study

Rendleman and Carabini (1979) develop a model to test for the efficiency of the Treasury bill futures market. The equilibrium price of a futures contract is determined on the basis of arbitrage relationships between the futures contract and the spot bills. The model is then expressed in the form of the IMM Index and transactions costs are considered explicitly. These give rise to a range of equilibrium IMM Index values within which arbitrage is not possible.

To test for efficiency, Rendleman and Carabini examine the relationship between observed IMM Index values and the theoretical values. They use daily data and the analysis is limited to the first three contracts for similar reasons as those cited by Capozza and Cornell. In the first stage of the analysis, the actual IMM Index values are compared to their theoretical counterparts assuming no transactions costs. They find that there are generally positive price differences for the nearby contract and negative differences for longer term contracts. (Notice that this is consistent with Capozza and Cornell's findings since prices and yields are inversely related.) Rendleman and Carabini annualize the spread differentials to make them comparable over time, because, as they point out, arbitrage profits in the short-term contracts can be earned in a shorter period of time than those in the long-term contracts.

In the second stage of their analysis, the researchers account explicitly for the costs associated with commissions and initial margins, and under these conditions, only one-third of the observations offer arbitrage opportunities. Rendleman and Carabini conclude that there exist quasi-arbitrage possibilities in the short-term contracts but that they do not appear to be large enough to offer attractive investment alternatives to the short-term portfolio manager.

In their analysis, Rendleman and Carabini consider the explicit costs of commissions and initial margin. In doing so, they conclude that differences that appear to offer arbitrage opportunities are not significant when the above costs are considered. Their analysis points out the importance of including all variables that affect the futures and forward rates.

Vignola and Dale Study

Taking a fresh approach to the question of the equilibrium price, Vignola and Dale (1979b) compare the actual futures price with two alternative specifications of the equilibrium futures price. Based on Working's (1949) theory of carrying charges, the first specification of the equilibrium futures price is an overnight cost-of-carry model where the equilibrium futures price equals the spot price of the cash bill deliverable on the contract plus the cost of storing that bill. The cost of storage is equal to the financing cost necessary to store the bill until the maturity data on the futures contract. This, in turn, is equal to the rate on a reverse repurchase agreement.

In the second specification, Vignola and Dale resort to the familiar approach of comparing the actual price of the futures contract to the price of a forward contract constructed on the basis of cash Treasury bills. Using the mean of the bid-ask spread, and the federal funds rate as a proxy for the repurchase rate, they calculate two daily time series of differences; a series between the actual futures price and the equilibrium cost-of-carry futures price and the other between futures and forward prices.

Based on their analysis of the annualized differences between the actual futures prices and the two alternative specifications, Vignola and Dale conclude that the cost-of-carry model is more appropriate for the analysis of pure arbitrage because it embodies the concept of the opportunity cost of funds in the repurchase rate. On the other hand, they contend that the forward rate model is appropriate for analysis of quasi arbitrage since the forward rate may be obtained from a combination of cash bills held in an existing portfolio.

This study is unique in its cost-of-carry model, an approach which deserves further study. The forward rate model, however, ignores the margin account of the futures transactions and possible influence it may have on the relationship between the futures and forward rates.

Morgan Studies

In two separate articles, Morgan (1979, 1980) points out that one of the major differences between forward and futures contracts is the insurance role provided by the clearing house as a result of its requirement that clearing members resettle their accounts each day through the process of marking to market. Recognizing the contributions of Black (1976) in the pricing of commodity contracts, Morgan contends that due to marking to market, futures prices will not equal forward prices even in an efficient market. Specifically, he shows that while the change in the value of a futures contract is equal to the change in the contract's price, the change in the value of a forward contract is equal to the discounted change in the forward price. Under the assumption that the forward and futures contracts are equivalent with respect to all other characteristics, the change in the value of each of the contracts would be the same. If futures and forward prices are equal at the beginning of a period, the above condition implies that the two sets of prices must be different at the end of the period if value changes during the period are to be equal. Morgan concludes his analysis by stating that the difference between futures and forward prices arise because futures prices incorporate expectations regarding the course of interest rates between the initiation and delivery dates of the contract whereas forward prices do not.

Morgan's studies are the first to recognize the differences between futures contracts and forward contracts. The process of marking to market the daily price changes of futures contracts has significance for the analysis of futures and forward rate because, as he points out, interest may be earned (foregone) on the cash withdrawals (margin calls) from the margin account. The expectation of interest earned (foregone) must be considered in any ex-ante calculation of arbitrage returns.

Arak and McCurdy Study

In addition to providing a general description of the Treasury bill futures market, Arak and McCurdy (1980) investigate tax implications on futures prices and how taxes might affect the spread between futures and forward prices. They point out that there are two tax factors that provide the incentive to use the markets to reduce tax liabilities. The first is the fact that the IRS assumes that a gain or loss on a futures contract is realized only when the futures contract is closed out (not over time in the daily resettlement process) and the second is that the IRS treats a gain or loss on a long futures position that is closed out more than six months after it is initiated as a long-term capital gain or loss. On the other hand, any gain or loss on a short futures position is considered a short-term capital gain or loss no matter how long the position remains open. Arak and McCurdy postulate that investors would prefer to have price appreciation treated as a long-term capital gain rather than ordinary income. Given this, some who might normally purchase, say, 52-week bills would have the incentive to buy distant futures contracts instead and, as they matured, sell them off to take their capital gains, and invest their funds in 3-month bills. This kind of activity would reduce the demand for 52-week bills thereby raising their discount rates, increase the demand for distant futures, decreasing their rates, and the net effect would result in a downward bias of the discount rates on futures contracts relative to the implicit forward rate on cash bills.

The study by Arak and McCurdy, although it does not specifically address the question of market efficiency or the price relationship between the futures and the cash market, does introduce the tax variable as a significant motivation for certain behavioral patterns and observed relationships. The significance of this variable with respect to the price structure between the futures and cash markets needs to be studied further.

Summary and critique of studies

In one way or another, each of the articles reviewed above attempts to analyze the relationship between futures and cash prices (rates). This does not imply that each has the same objective; whereas, Emery and Scott are concerned with testing for the validity of the expectations hypothesis of the term structure, Rendleman and Carabini use similar procedures to test for the efficiency of the Treasury futures market. Despite some variety in their objectives, most of the papers are essentially concerned with the hypothesis that the futures rate on a contract is equal to the forward rate implied in the cash term structure of interest rates. For the most part, it is the construction of the arbitrage model and treatment of transactions costs, compounding periods, annualization periods, contract maturities, and rate approximation formulas which distinguish one methodology from the next. For example, Poole's analysis was based on an arbitrage model of the equilibrium futures rate including commissions and initial margins costs, whereas Vignola and

Dale analyzed the actual futures price abstracting from explicit consideration of transactions costs.

Despite the conflicting evidence cited by the various researchers, in general, the futures rates are found to be biased downward relative to the forward rate for the nearby contracts and biased upward for the more distant contracts. Even when the methodology is similar, the studies differ with respect to their conclusions. For example, the studies by: (1) Poole, and Lang and Rasche, and (2) Puglisi, and Vignola and Dale exhibit completely different conclusions, although they are quite similar in structure.

With the exception of Morgan, each of the authors (Lang and Rasche, Puglisi, Vignola and Dale, Copozza and Cornell, and Rendleman and Carabini) whose empirical analysis indicated statistically significant differences between the futures and the forward rates, conclude that the Treasury bill futures market is inefficient. Various reasons are cited to explain the futures-forward bias including the risk of a futures contract compared to a forward contract, the newness of the market and the lack of use by institutional investors. However, the major reason cited by nearly all of the authors to explain the bias is the existence of fixed transactions costs, i.e., commissions and initial margins. Although the existence of these transactions costs does seem to prevent pure arbitrage, it does not fully explain why quasi arbitrage profits continue to exist, nor does it explain the observed pattern of the bias between futures and forward rates. Perhaps this suggests that not all of the relevant variables have been adequately accounted for.

The conflicting evidence regarding the efficiency of the Treasury bill futures market and the observed bias between the actual and equilibrium or theoretical futures price tends to suggest that a number of issues concerning model specification remain unresolved. In fact, as Morgan points out, it is not entirely clear that even in an efficient market, the futures rate should equal the forward rate.

A major difference between the futures and forward contracts is the daily resettlement process of marking to market. Although futures and forward contracts may be similar in other respects, the inter-temporal distinction in the way in which profits or losses are incurred may account for some of the difference in prices between the two kinds of contracts. Specifically, since profit withdrawals or margin calls on a futures contract depend on the time series of futures prices during the open contract, the equilibrium price of a futures contract may be influenced by investor expectations of futures prices over time. Since by their very nature, expectations are uncertain, the equilibrium futures price may also embody a risk premium in compensation for the risk encountered in the margin account.

CHAPTER IV. THE STRUCTURE OF PRICES AND YIELDS BETWEEN THE FUTURES MARKET AND CASH MARKET

To alleviate some of the shortcomings of existing models, a specification of the equilibrium futures price is developed which recognizes all of the variables that are relevant to trading futures and forward contracts. These include commissions, initial margins and risk premiums. The relationship between observed futures rates and forward rates is explored on the basis of these variables.

No Transactions Costs

The time dimension of a 90-day Treasury bill futures contract is represented in Figure 4.1. An investor, by going long or short on a Treasury bill futures contract, may contract at time 0 to buy or sell Treasury bills. Although the commitment price of the bills is set at time 0, the bills are not delivered or accepted until the maturity date of the futures contract, time m. At time m, delivery or acceptance of the Treasury bills is executed, and 91 days thereafter, at time n, the bills mature. In accordance with Figure 4.1, let P_m and P_n equal the time 0 market prices per \$100 of par value of spot market Treasury bills with maturity dates at time m and n, respectively. Based on discount rates and a 360 year, the prices of the spot bills, P_m and P_n, may be expressed as:

$$P_m = 100 - r_m\left(\frac{m}{360}\right)100 \tag{4.1a}$$

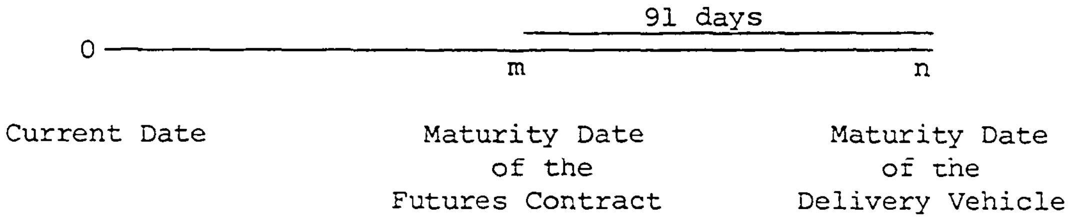

Figure 4.1. Time dimension of a Treasury bill futures contract

and

$$P_n = 100 - r_n(\frac{n}{360})100 , \qquad (4.1b)$$

where r_m and r_n are the discount rates on the spot market Treasury bills which mature at time m and n, respectively. In addition, let FP represent the time 0 market price of a futures contract per \$100 of par value at maturity, time m.

Suppose that at time 0, an n-day bill can be purchased (or sold) at a price P_n. At time n, this bill will have a maturity value of \$100 and the dollar return from holding such a bill will be equal to:

$$R_A = (100-P_n) , \qquad (4.2a)$$

the dollar discount. Alternatively, suppose that instead of purchasing the n-day bill and holding it to maturity, a simultaneous purchase consisting of the m-day bill and a futures contract is executed. In this tandem transaction, the m-day bill is purchased at time 0 and it matures at time m. At time m, the maturity value of this bill is used to fund the settlement of the futures position, the price, FP, which was established at time 0. If FP is less than $100, the maturity value of the m-day bill, then it is necessary to purchase only a fractional amount, $(\frac{FP}{100})$, of the m-day bill in order to fund the futures position at time m. Since the Treasury bills accepted on the futures contract have a time n maturity value of $100, the total dollar return, R_B, from the tandem purchase of the m-day bill and the futures contract will be:

$$R_B = [100 - (\frac{FP}{100})P_m]. \tag{4.2b}$$

In this example, the holding period, 0-n, is the same regardless of which investment alternative is chosen--the purchase of the n-day bill or the simultaneous purchase of the m-day bill and the futures contract. If investors are indifferent to all characteristics other than the holding period yield of the alternatives available to them, competitive market forces will ensure parity between the holding period returns of the two alternative investments, that is:

$$R_A = R_B. \tag{4.2c}$$

Similarly, consider the holding period 0-m. Within this period, the investor can purchase either the m-day bill at P_m and hold it to maturity or he can simultaneously purchase the n-day and sell a futures contract with the intent of delivering the n-day bill against the futures contract, at time m. The 0-m holding period return of the simultaneous purchase of the n-day Treasury bill and sale of the futures contract will equal:

$$R'_B = (FP - P_n). \tag{4.3a}$$

Because the m-day bill will yield $100 at its maturity, only a fractional amount, $(\frac{FP}{100})$, of that bill need be purchased in order for it to yield FP at its maturity, making the maturity value comparable to the first alternative. Therefore, the holding period return of the m-day bill is expressed as:

$$R'_A = [FP - (\frac{FP}{100})P_m]. \tag{4.3b}$$

As with the 0-n holding period, market forces will ensure parity between the returns of each alternative with a 0-m holding period; therefore,

$$R'_A = R'_B. \tag{4.3c}$$

In the absence of transactions costs, market forces in either holding period, 0-m or 0-n, will ensure that $R_A = R_B$ or $R'_A = R'_B$. From Equations 4.2c and 4.3c, it can be demonstrated that equilibrium will be obtained only when:

$$P_m(\frac{FP}{100}) = P_n \tag{4.4a}$$

or

$$FP^* = 100(P_n/P_m). \tag{4.4b}$$

Equation 4.4b defines the equilibrium price, FP*, of the futures contract in terms of the time 0 spot market prices of the m- and n-day bills. If the observed futures price is equivalent to the theoretical futures price as described above, risk-free arbitrage opportunities will not be available.

The equilibrium futures price, FP*, is illustrated in Figure 4.2. The vertical axis represents the price of the futures contract and the horizontal axis represents time. In relation to Figure 4.1, time m, the maturity date of the futures contract, is represented in Figure 4.2 by the intersection of the horizontal and vertical axes. Points to the right of this intersection indicate the price of an open futures contract, that is, at a point in time prior to the contract's maturity. Thus, the line drawn in Figure 4.2 portrays a time series of equilibrium futures prices.

Suppose, however, that the existing time 0 futures price were represented by point A. Because point A is below the equilibrium futures price, there would exist an incentive to take advantage of arbitrage profits by exchanging the n-day bill for the equivalent position of an m-day bill and long futures contracts. The latter position would yield a higher rate of return than the holding period of the n-day bill.

To illustrate this kind of arbitrage, suppose that the time 0 prices for the 90- and 180-day spot bills were 97.50 and 96.00, respectively. These prices would imply discount rates of 10 percent for the 90-day bill and 8 percent for the 180-day bill. According to

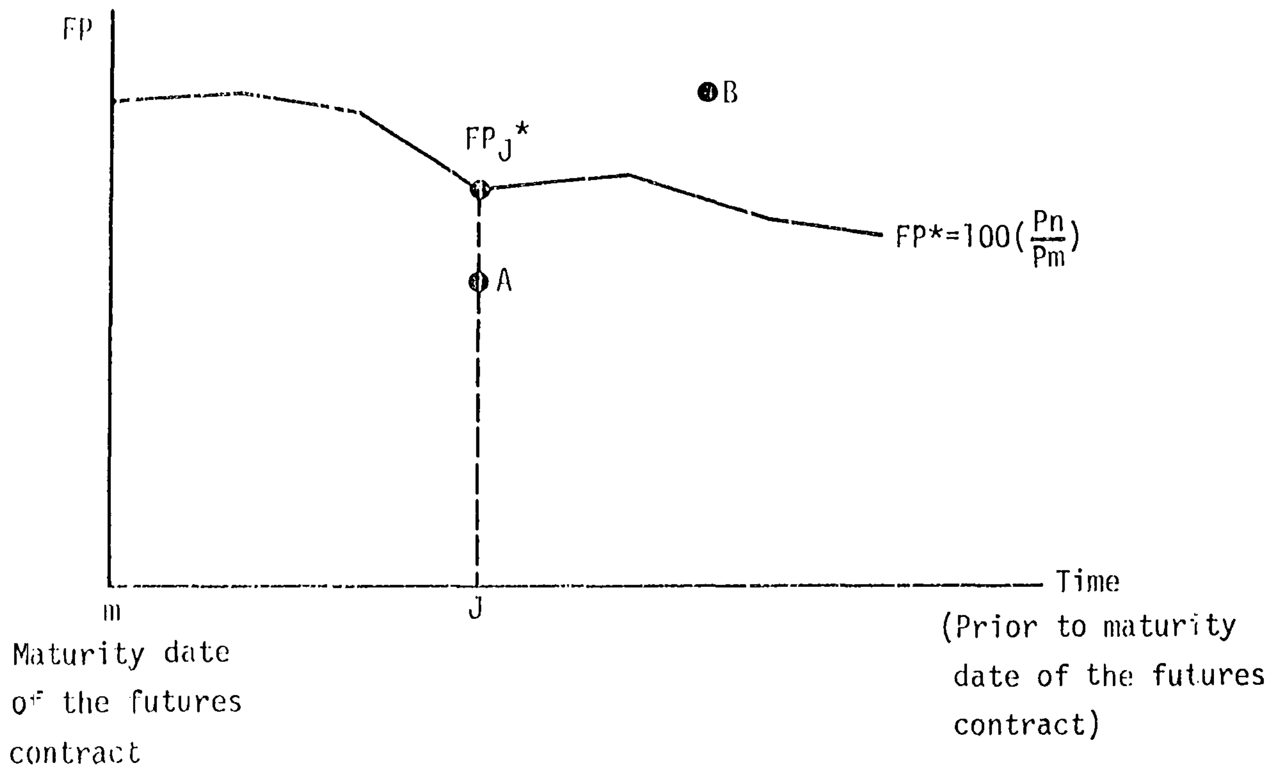

Figure 4.2. Equilibrium futures price curve

Equation 4.4b, the equilibrium futures price would be 98.46, implying a discount rate of 6 percent. Under these circumstances, the same holding period return would be earned over 180 days regardless of the investment chosen. If the 180-day spot bill were purchased and held to maturity, the holding period yield would be 8 percent. If, on the other hand, a simultaneous purchase consisting of the 90-day bill and a futures contract were executed, the average return over 180 days would also be 8 percent. Suppose, however, that the futures price were 98.35, below the equilibrium futures price. This would imply a discount rate of 7 percent on the contract and 8.5 percent over 180 days, when combined with the purchase of the 90-day bill. This would be a higher rate of return than the 8 percent that could be earned by purchasing the 180-day bill and holding it to maturity.

Similarly, an observed futures price represented by point B would create an incentive to substitute for the m-day bill a tandem position composed of an n-day bill and a short futures contract. Although the holding period, 0-m, remains the same, the returns from arbitraging the market by taking a short futures position are positive. Thus, it is clear that points A and B represent disequilibrium prices, thereby indicating the existence of profitable arbitrage opportunities. Points below the equilibrium level of futures prices provide the incentive to be long in the Treasury bill futures market while prices above the equilibrium level signal a short futures position. Having characterized efficiency in the Treasury bill futures market by a lack of arbitrage opportunities, points A and B as well as all other points off of the equilibrium line in Figure 4.2 must be regarded as observed prices in an inefficient market.

Transaction costs

The relationship expressed in Equation 4.4b is altered when transactions costs are considered in the analysis. Indeed, when transactions costs are included, profitable arbitrage may not exist even though the observed futures price does not equal the theoretical futures price, FP*.

The cost of arbitrage between the cash and futures market for Treasury bills may be divided into two categories; those incurred in the spot market and those incurred in the futures market. If pure arbitrage is undertaken, spot market costs consist of: a) the costs of financing a short position in the spot market, and b) the transactions costs associated with buying or selling Treasury bills in the spot market. Quasi arbitrage, on the other hand, involves only the transcations costs associated with buying and selling from an existing portfolio, that is, the bid-ask spread.

Futures market costs consist of the commission charges, the initial margin, and any variation margin (margin calls). With the inclusion of transactions costs in the analysis, Equation 4.4b will be altered such that there will be a range, rather than a single value, of equilibrium futures prices where arbitrage is not pfofitable.

Following Poole's (1978) lead in the analysis of the arbitrage range, the lower and upper future's prices of that range are developed.

The exposition of the arbitrage range is developed in terms of quasi arbitrage. Therefore, to determine the lower futures price of the arbitrage range, suppose that the n-day spot bill is owned. If the price of a futures contract is low enough, it will pay the investor to substitute a portion of the n-day bill already owned with an equivalent investment consisting of a purchase of a m-day bill and a futures contract. The holding period of each of these alternatives is 0-n and the maturity value of each is $100. Therefore, if the price of the futures contract is low enough that less than 100 percent of the n-day bill must be sold in order to fund the simultaneous purchase of the m-day bill and the futures contract, the remaining fraction of the n-day bill still owned would constitute an arbitrage profit. This assumes, of course, a hold-to-maturity strategy where the returns on the alternatives must be compared over a similar holding period.

The n-day bill may be sold at the dealer's bid price:

$$P_n^b = 100 - r_n^b\left(\frac{n}{360}\right)100. \tag{4.5a}$$

The m-day bill must be purchased at the dealer's ask price;

$$P_m^a = 100 - r_m^a\left(\frac{m}{360}\right)100. \tag{4.5b}$$

The bid yield on the n-day bill, r_n^b, and the ask yield on a m-day bill, r_m^a, are annualized discount rates and are based on a 360 day year. The bid-ask spread represents the transactions costs of doing business in the spot market. Enough m-day bills must be purchased so that their maturity value will be sufficient to fund the settlement on the futures contract when it matures at time m. The price of a futures contract, FP, established at time 0 is:

$$FP = 100 - r^f(\frac{91}{360})100, \qquad (4.6)$$

where r^f is the annualized Treasury bill discount yield on the Treasury bills specified by the futures contract.

Fixed Transactions Costs

In addition to the price of the futures contract, other associated costs must be considered; commissions and margin requirements.

Commissions

The normal commissions charge on a round turn futures transaction is $60.00 per contract. This is usually paid when the contract is liquidated, either by offset or delivery. Although commission charges are now negotiable, the flat fee usually remains at $60.00, unless the number of contracts traded is very large. The commission charge is also assessed on a basis which abstracts from the holding period of the open contract. In other words, with the exception of day trades, the commission is the same regardless of the holding period of the open contract.

Initial margin In addition to the commission charge, margin money must be posted by both long- and short-position traders. The amount of initial margin required to be posted has changed over time and is dependent on the type of position taken in the futures market. Generally, the initial margin has equaled \$1500 per contract.

The fixed costs of trading futures contracts must be taken into account in any ex ante calculation of arbitrage profits. Letting c equal the commission charge per \$100 of futures contracts and d equal the initial margin on a like basis, the total cost of settling the long futures position at time m is:

$$(FP + c - d). \qquad (4.7a)$$

This expression shows explicitly that at time m, the purchase price, FP, and the contract commission, c, must be paid in order to settle the long position. The initial margin, d, however, is returned to the investor when the contract obligations are fulfilled unless, of course, there is a net debit to be accounted for as a result of margin calls not yet met. In order to fund the futures position, enough m-day bills must be purchased at price P_m^a to provide (FP + c - d) at their maturity. Therefore, the total amount of cash needed at time 0 to purchase the m-day spot bill is:

$$\left(\frac{FP + c - d}{100}\right) P_m^a + d \, , \qquad (4.7b)$$

where it is shown explicitly that the initial margin, d, is paid at time

0 when the futures contract is purchased and returned to the investor, at time m, when it is closed out.

At time n, the Treasury bills which are accepted on the futures contract will have a maturity value of $100. Therefore, over the holding period, 0-n, the total dollar return from the simultaneous purchase of the m-day spot bills and the futures contract will be:

$$R_A = 100 - [(\frac{FP + c - d}{100})P_m^a + d]. \quad (4.8)$$

Because none of the variables in Equation 4.8 is random at the time the arbitrage is initiated, the return, R_A, is known with certainty, and no risk is involved. If, on the other hand, the n-day bill is held in the portfolio until maturity at time n, the return will be:

$$R_B = (100 - P_n). \quad (4.9)$$

Setting Equations 4.8 and 4.9 equal to one another yields essentially the same results as expressed by Equation 4.4b.

The total cash requirement at time 0, represented by Equation 4.7b, is raised by selling a fraction, Z, of the n-day bill already owned. If that fraction is less than 100 percent, the remaining fraction, 1-Z, of the n-day bill still owned will represent an arbitrage profit. In other words, Z less than unity implies the existence of profitable arbitrage opportunities, and according to our previous definition, an inefficient market. If the ex ante calculation of arbitrage returns requires that Z is less than unity, then the total return R_T, from the arbitrage operation will be:

$$R_T = R_A + (1-Z)R_B = \{[100 - (\frac{FP + c - d}{100})P_m^a + d]$$

$$+ [(1-Z)(100-P_n^b)]\}. \qquad (4.10)$$

To raise the funds to purchase enough m-day bills that upon maturity will have a value sufficient to fund the futures position, ZP_n^b must be sold. If $ZP_n^b = (\frac{FP + c - d}{100})P_m^a + d$, then $[\frac{(\frac{FP + c - d}{100})P_m^a + d}{P_n^b}]$ is the fraction of n-day bills which must be sold. Arbitrage will be profitable only if that fraction is less than unity. Therefore, in terms of futures prices, arbitrage will be undertaken only if:

$$(\frac{FP + c - d}{100})P_m^a + d < P_n^b \qquad (4.11a)$$

or

$$FP < 100(\frac{P_n^b}{P_m^a}) - \frac{c}{100}P_m^a + \frac{d}{100}P_m^a - d. \qquad (4.11b)$$

If the observed futures price is lower than the term on the right hand side of the inequality, quasi arbitrage will be profitable.

Given the well-known fact that yields and prices of marketable interest bearing securities are inversely related, the above relationship may be expressed in the form of the discount rate. By substituting Equations 4.5a, 4.5b, and 4.6 in Equation 4.11b, the arbitrage relation can be expressed as follows:

$$r^f > \frac{[r_n^b(\frac{n}{91}) - r_m^a(\frac{m}{91}) - \frac{c}{100}r_m^a(\frac{m}{91}) + \frac{d}{100}r_m^a(\frac{m}{91}) + \frac{c}{100}(\frac{360}{91})]}{[1 - r_m^a(\frac{m}{360})]}. \qquad (4.12)$$

Equation 4.12 is an expression of the upper arbitrage limit in terms of the discount rate per \$100 of futures contracts. If the yield on the futures contract, r^f, is higher than the term on the right hand side of the inequality, quasi arbitrage profits may be obtained by selling a fraction of the n-day bill from the existing portfolio and simultaneously purchasing the m-day spot bill and a futures contract. The costs of conducting the above transactions are shown explicitly in Equation 4.12. The bid-ask spread of the spot market transactions is represented by the bid-ask yields and the futures market commission and margin requirements are reflected by the terms $\frac{c}{100}r^a_m(\frac{m}{91})$, $\frac{d}{100}r^a_m(\frac{m}{91})$ and $\frac{c}{100}(\frac{360}{91})$.

In a similar fashion to the foregoing development, the upper (lower) arbitrage price (yield) may be obtained. If the futures price (yield) is high (low) enough, it will be profitable to substitute the n-day bill and a short futures position for a portion of the m-day bill held in the portfolio. In this case, the holding period is 0-m, not 0-n, as before. If the fraction of the m-day bill sold to finance the purchase of the n-day bill and the short futures position is less than unity, the remaining fraction of the m-day bill left in the portfolio after all of the arbitrage transactions have been completed represents a quasi arbitrage profit. The upper arbitrage price may be expressed analogously to Equation 4.11b as:

$$FP > 100(\frac{P^a_n}{P^b_m}) + \frac{c}{100}P^b_m - \frac{d}{100}P^b_m + d. \qquad (4.13)$$

If the observed futures price is above the expression on the right hand side of the inequality, quasi arbitrage profits will be available by selling the m-day spot bill while simultaneously purchasing the n-day spot bill and going short in the futures market.

By substituting Equations 4.5a, 4.5b, and 4.6 into Equation 4.13, the lower arbitrage limit in terms of the discount rate can be expressed as:

$$r^f < \frac{[r_n^a(\frac{n}{91}) - r_m^b(\frac{m}{91}) + \frac{c}{100}r_m^b(\frac{m}{91}) - \frac{d}{100}r_m^b(\frac{m}{91}) - \frac{c}{100}(\frac{360}{91})]}{[1 - r_m^b(\frac{m}{360})]} . \quad (4.14)$$

If the observed yield on a futures contract that matures at time m is lower than the expression on the right hand side of the inequality, quasi arbitrage is possible by selling a fraction of the m-day bill and substituting it with enough n-day bills to satisfy delivery on a short futures position.

Combining Equation 4.11b and 4.13 defines the no-arbitrage range of futures prices:

$$[100(\frac{P_n^b}{P_m^a}) - \frac{c}{100}P_m^a + \frac{d}{100}P_m^a + d < FP^* < 100(\frac{P_n^a}{P_m^b}) + \frac{c}{100}P_m^b - \frac{d}{100}P_m^b + d] . \quad (4.15)$$

Likewise, from Equations 4.12 and 4.14, the no-arbitrage range of the futures yield may be defined as:

$$\frac{[r_n^a(\frac{n}{91}) - r_m^b(\frac{m}{91}) + \frac{c}{100}r_m^b(\frac{m}{91}) - \frac{d}{100}r_m^b(\frac{m}{91}) - \frac{c}{100}(\frac{360}{91})]}{[1 - r_m^b(\frac{m}{360})]} < r^{f*}$$

$$< \frac{[r_n^b(\frac{n}{91}) - r_m^a(\frac{m}{91}) - \frac{c}{100}r_m^a(\frac{m}{91}) + \frac{d}{100}r_m^a(\frac{m}{91}) + \frac{c}{100}(\frac{360}{91})]}{[1 - r_m^a(\frac{m}{360})].} \quad (4.16)$$

Having defined efficiency in terms of the potential for arbitrage between the futures and the spot market for 90-day Treasury bills, Equations 4.15 and 4.16 show the theoretical range of prices and yields of futures contracts in an efficient market. If the observed futures price fluctuates beyond the arbitrage limits developed above, the investor with a portfolio of m- and n-day Treasury bills can improve the rate of return on his portfolio by altering the composition of his spot market securities and going short on long in the futures marekt. Thus, an inefficient market presents an attractive means for the portfolio manager to increase returns while maintaining a specified risk exposure.

The existence of transactions costs give rise to a range of futures prices (yields) for which arbitrage opportunities are not available. This range of futures prices is illustrated in Figure 4.3, where, as in Figure 4.2, futures prices are measured along the vertical axis and time is represented by the horizontal axis. The upper critical futures price, FP^U, and the lower critical futures price, FP^L, define the extremities between which arbitrage profits are

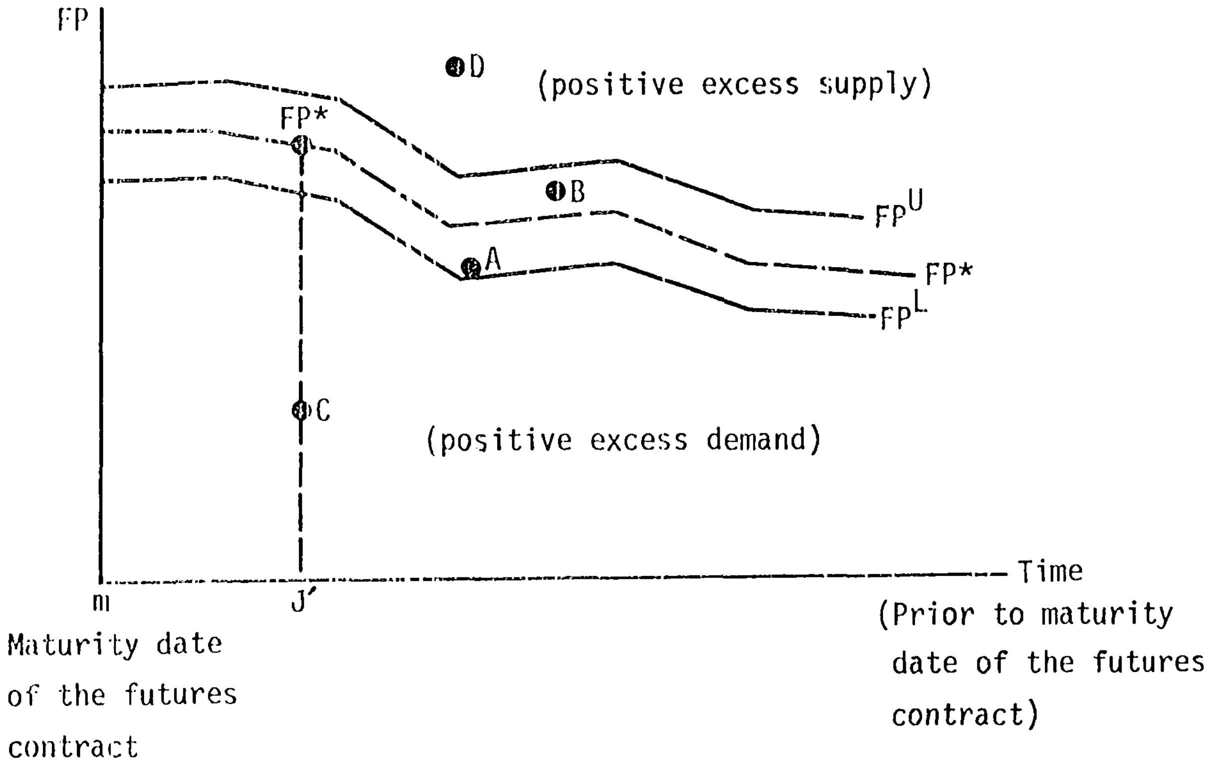

Figure 4.3. Equilibrium futures price range

not available. For example, consider point A. If there were no fixed costs associated with futures trading, point A would represent a disequilibrium situation which could be profitably arbitraged by going long in the futures market. In the absence of transactions costs, the arbitrage profits in this case would be a function of the spread between the observed futures price, point A, and the equilibrium futures price, FP*. However, the fact that the futures price represented by point A is within the no-arbitrage range indicates that the fixed costs of trading futures outweigh the potential gain due to the observed spread. A similar conclusion would hold for point B where normally (abstracting from transactions costs) the existence of potential arbitrage profits would dictate a short futures position. Poole's analysis considers only the fixed costs of futures trading. Therefore, in Figure 4.3, the spread of the arbitrage band surrounding the equilibrium futures price is defined solely by the costs associated with commissions and initial margin.

To extend Poole's analysis, variable costs are introduced and their effect on arbitrage and the equilibrium futures price is analyzed. The variable cost of futures trading is associated with the process of marking to market, which is the daily resettlement procedure occurring in the margin account.

Variation margins

To protect the investor against accumulated losses and to ensure the performance of the contract obligations, the IMM specifies a level called the "maintenance margin" or "variation margin", below which the initial margin is not permitted to float. As described in Chapter II, a balance is calculated by debiting or crediting the daily price movements of a contract to the initial margin. If price movements are favorable to the contract holder, then surplus funds of the margin balance can be withdrawn. If, however, they are unfavorable, margin calls must be met. Everyday, the IMM debits or credits the balance of a clearing member's account by the dollar change in the value of his open contracts resulting from the price movements occurring in the previous trading session.

Assuming price movements, the total amount of cash which must be made in margin calls (or withdrawn) is a function of the difference between the settlement price at the time of contract liquidation or delivery and the price at which the futures position was opened. Letting V represent the variation margin, the total gain or loss per contract to the margin account may be expressed as:

$$V = [FP_m - FP](k), \tag{4.17}$$

where FP_m is the contracts ending settlement price, FP is the price at which the futures position was opened and k is the dollar equivalent of each basis point change in the contract's price, i.e., $25.00. Thus, unlike the fixed costs of commission or initial margin, the cost or gain associated with variation margin is a function of the change in futures prices over time and the number of contracts held.

Because FP_m is not observable at the time the contract is initiated, the investor must estimate what he believes will be the settlement price of the contract upon delivery. It has been assumed throughout the preceeding analysis that the arbitrage strategies call for the satisfaction of long or short futures contracts by actual delivery or acceptance. Thus, the investor who decides to arbitrage is assured that upon completion of his contract obligations, both the initial margin and the variation margin would be returned to him. This would seem to make the necessity of estimating the contract's settlement price at maturity a moot point. However, margin calls must be made in cash, and though the variation margin is effectively returned to the investor at the contract's maturity, the interest earned or foregone on that money is not. Therefore, the investor will anticipate an interest loss or return of:

$$\tilde{Vr}^e = [\tilde{FP}_m^e - FP](k)(r), \qquad (4.18)$$

where $\tilde{FP}_m^e$, a random variable signified by the tilde, is the anticipated settlement price of the futures contract and r is the prevailing rate of interest or opportunity cost of short-term funds. $\tilde{Vr}^e$, also a random variable, is the anticipated interest cost (return) on the margin account over the life of the contract and is incurred (earned) in addition to the stated rate of the contract. On a per contract basis, $\tilde{Vr}^e$ is dependent on the investor's forecast of futures prices over time and short term interest rates.

Various implications follow from the consideration of the margin account. First, the cost (gain) associated with the variation margin cannot be estimated with certainty if the time series of futures prices and interest rates is not certain. If an investor is long in futures and the price of his contracts rises (falls), then there will be an interest gain (loss) on the margin money withdrawn (posted) on his account. Expectations of the time series of futures prices will determine the anticipated interest cost or gain on the margin account. Since ex ante calculations of arbitrage profits must account for all costs, fixed as well as variable, the uncertain variable cost (return) associated with the variation margin means that, strictly speaking, arbitrage involving the futures market is not totally risk-free. In so far as Equation 4.8 is concerned, this suggests that R_A is in fact a random variable, not a certain return. As a result, investors or arbitragers may demand that a risk premium be included into the futures return in order to compensate for the risk incurred in the margin account.

A second implication results from the effect that competitive market forces will have on disequilibrium futures prices over time and the behavioral expectations generated from changing prices. Consider point C in Figure 4.3. If the futures price at time J' is below the range of equilibrium futures prices, as point C is, then quasi arbitrate profits may be obtained by selling the n-day bill and simultaneously purchasing the m-day bill and going long in the futures market. Therefore, point C represents a disequilibrium price that will encourage positive excess demand for long futures contracts. The excess demand will tend to drive the futures price upwards, towards its equilibrium level. Thus, the investor who goes long in futures contracts at point C will anticipate a rise in the price of his contract over the holding period. Assuming his expectations materialize, the investor's equity position in the margin account will be enhanced, entitling him to withdraw cash from his margin balance. The cash profits may then be invested to earn a positive rate of return at prevailing interest rates. Point C, then, represents a disequilibrium futures price that will stimulate an excess demand for long futures and the anticipation of a net interest gain on the margin account.

Similarly, if the observed futures price were at point D in Figure 4.3, arbitrage would dictate going short in the futures market. In this case, an excess supply of short futures contracts would depress contract prices over time. A short futures position coupled with falling prices would also show a net gain on the margin account upon which interest could be earned. Therefore, competitive market forces at point D as well as point C would stimulate favorable price movements for the contract holder with the result that his margin account would be enhanced.

This analysis suggests that each investor who arbitrages the futures market would anticipate receiving a positive interest return on his margin account over the holding period. In other words, competitive market forces give rise to behavioral expectations such that $\tilde{V}r^e$ is positive. A distinction must be made, however, between the behavioral expectations of individual investors and mathematical expectations of the market place.

An investor's behavioral expectation concerning $\tilde{V}r$ depends upon his personal forecast of the ending settlement price of his futures contract, $\tilde{FP}_m$. Investors who arbitrage the market will expect $\tilde{V}r$ to be positive. In the mathematical sense, however, the expected value of $\tilde{V}r$ will equal the mean of the distribution of all possible values for $\tilde{V}r$. From Equation 4.18, it is evident that the random variable $\tilde{V}r$ is a function of the random variable $\tilde{FP}_m$. Thus, the expected value of $\tilde{V}r$ will depend upon the mathematical expectation of $\tilde{FP}_m$. From Chapter II, it will be recalled that a major proposition of the efficient market hypothesis states that the mathematical expectation of next period's price is identical to the observed price in the current period. By iteration, it logically follows that in any period, the expected price n periods in the future will also equal the currently observed price. Thus, in terms of the efficient market hypothesis, the mathematical expectation of $\tilde{V}r$ must be zero. The distinction concerning expectations should now be clear. Any individual investor may reasonably anticipate a positive value of interest earned on the margin account, $\tilde{V}r^e > 0$; however, in a mathematical sense, the efficient market hypothesis suggests that the expected value of interest in the margin account considered over all investors is equal to zero, $E(\tilde{V}r) = 0$.

Holding Period Returns

The futures market

When all fixed and variable transactions costs are accounted for in the ex ante consideration of the futures position, the holding period gain of the contract will be composed of the futures price established at time 0, FP; transactions costs including commissions, c, and initial margin, d; and investor expectations concerning the return on the margin account, $\tilde{V}r^e$. By substituting for the variables above, an expression similar to Equation 4.8 may be derived. The following shows the dollar return, $\tilde{R}$, per contract from the purchase of the m-day bill and the futures position when all costs are included:

$$\tilde{R} = \{[100 - ((\frac{FP + c - d}{100})P_m^a + d)] + [(\tilde{F}P_m^e - FP)(k)(r)]\}. \quad (4.19\text{-}4.20)$$

Unlike Equation 4.8 that consists of a certain return only, Equation 4.19-4.20 is composed of two parts: a nonrandom component enclosed in the first set of brackets and a random component enclosed in the second set of brackets. Hence, the total return, $\tilde{R}$, is a random variable. The nature of the returns may be shown explicitly by grouping random variables from nonrandom variables. Letting the constant $A = [100 - (\frac{FP + c - d}{100})P_m^a + d)$, the total return is:

$$\tilde{R} = A + [\tilde{F}P_m^e - FP)(k)(r), \quad (4.21)$$

where again the first term is nonrandom and $[(\tilde{F}P_m^e - FP)(k)(r)]$ is a random variable. Because the total return from arbitrage is uncertain, the futures position must be regarded as a risky prospect. This suggests that investors or arbitrageurs will demand a risk premium to compensate them for bearing the risk of their futures position that arises in the margin account.

An expression for this risk premium may be derived by considering the futures position in light of the expected utility hypothesis. By the expected utility hypothesis, a risk averter will, by definition, prefer a certain income to a random income taking on the values $(Y_o - x)$ and $(Y_o + x)$ with probabilities summing to unity, where Y_o equals certain wealth and x is the amount that can be won or lost by engaging in the risky prospect. In the present text, x corresponds to the interest earned (or foregone) on the margin account. Assuming that investors and arbitrageurs are risk averse and that their initial wealth position is Y_o, then their ending wealth position after arbitrage will be:

$$(Y_o + \tilde{R}) = [Y_o + A + (\tilde{FP}_m^e - FP(k)(r)]. \qquad (4.22)$$

The total wealth position is uncertain; however, by employing the expected utility hypothesis, a risk premium can be determined that will make the investor or arbitrageur indifferent between receiving a certain income or the variable income of the futures position.

Mathematically, the expected utility hypothesis may be expressed as:

$$U[E(Y_o + \tilde{R})] > E[U(Y_o + \tilde{R})] \qquad (4.23)$$

where U stands for the (undetermined) utility function, E is the expectations operator (in the mathematical sense) and Y_o and $\tilde{R}$ are as previously defined. Defining the risk premium to equal θ and incorporating it into Equation 4.23 results in an equilibrium condition that equates the utility of a certain income to that of a risky income,

$$U[Y_o + E(\tilde{R}) - \theta] = E(U(Y_o + \tilde{R})]. \tag{4.24}$$

Equation 4.24 specifies the condition for indifference between a risky income as expressed on the right hand side of the equality and its certainty equivalent on the left. These concepts are illustrated in Figure 4.4, where the vertical axis represents total expected return, the horizontal axis represents the variance of total return, and the upward sloping indifference curve implies risk aversion. Utility is constant along the indifference curve. Point B represents a risky investment with an expected return of $\tilde{R}_B$. $\tilde{R}_B$ is a random return that corresponds to Equation 4.19-4.20. The variance, σ_B^2, represents the risk associated with the futures position. Point B, however, has a certainty equivalent point, R_{ce}, and the two, are related by the utility function. In terms of return, the difference between the uncertain return of $\tilde{R}_B$ and its certainty equivalent, R_{ce}, is the risk premium denoted by θ. With respect to Equation 4.24, point B is comparable to the expression on the right hand side of the equation and R_{ce} is equivalent to the left hand side. The utility of the certainty equivalent is related to the utility of the return of the

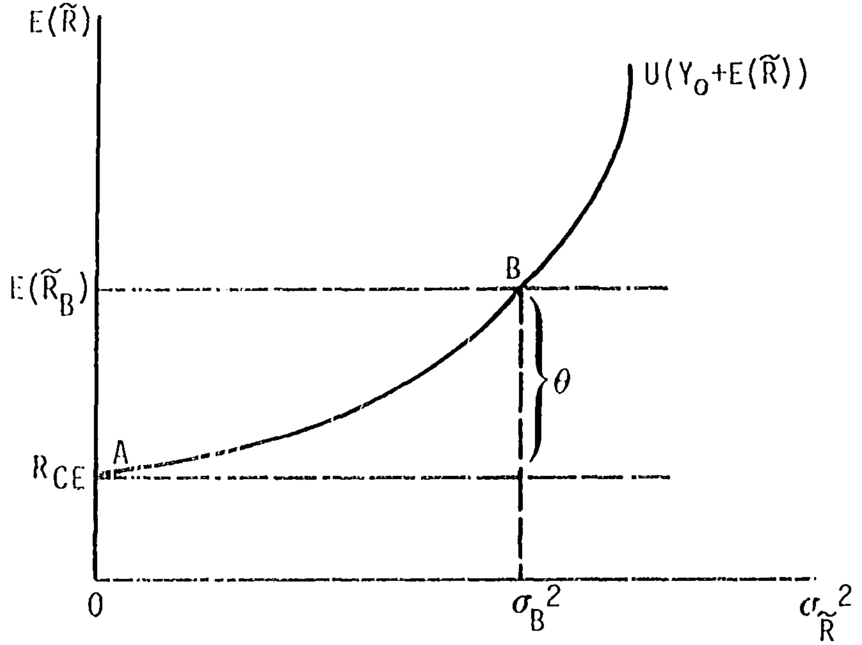

Figure 4.4. Utility in risk-return space

risky prospect by the risk premium, θ.

The risk premium that equates the two utilities may be derived by using a Taylor expansion on Equation 4.24, and then solving for θ. For any arbitrary function $\phi(x)$, the Taylor series may be used to approximate that function around a certain value, x_o. If $\phi(x)$ is continuously differentiable at x_o, then the function can be expanded around that point as follows:

$$\phi(x) = \frac{[\phi(x_o)}{0!} + \frac{\phi'(x_o)}{1!}(x-x_o) + \frac{\phi''(x_o)}{2!}(x-x_o)^2 + \ldots + \frac{\phi^n(x_0)}{n!}(x-x_o)^n] + R_n , \qquad (4.25)$$

where ϕ', ϕ'', ϕ''', etc. are the first, second and third derivatives of the function with respect to x (evaluated at x_o), the denominators are expressed in factorial notation and R_n is a remainder term included to represent the difference between $\phi(x)$, and the polynomial expansion of $\phi(x)$. The Taylor series expansion may be constructed to any order polynomial as long as appropriate derivatives exist. For example, a first order polynomial would constitute a linear approximation to the original function, a second order polynomial, a quadratic, a third order polynomial, a cubic, and so on.

With respect to Equation 4.24, the left hand side of the equation may be expanded around $[Y_o + E(\tilde{R})]$ in order to yield:

$$U[Y_o + E(\tilde{R}) - \theta] = U(Y_o + E(\tilde{R})) - \theta U'(Y_o + E(R)) + R_n \qquad (4.26)$$

Similarly, the right hand side of Equation 4.24 may be expanded around the mean of the random variable $Y_o + \tilde{R}$ with the following result:

$$E[U(Y_o + \tilde{R})] = U(Y_o + E(\tilde{R})) + \frac{U''(Y_o + E(\tilde{R}))}{2!} + \sigma_{\tilde{R}}^2 + R_n \qquad (4.27)$$

To satisfy the condition of indifference between utilities, the right hand side of Equation 4.26 is set equal to the right hand side of Equation 4.27:

$$U(Y_o + E(\tilde{R})) - \theta U'(Y_o + E(\tilde{R})) + R_n = U(Y_o + E(\tilde{R})) + \frac{U''(Y_o + E(\tilde{R}))}{2!}\sigma_{\tilde{R}}^2 + R_n \qquad (4.28)$$

and the risk premium may be solved for directly;

$$\theta = -\frac{1\,U''(Y_o + E(\tilde{R}))}{2U'(Y_o + E(\tilde{R}))}\sigma_{\tilde{R}}^2. \qquad (4.29)$$

The risk premium may be thought of as a bribe. It is the amount of money that the risk averter would demand in compensation for bearing the risk to obtain an uncertain income. In the present context, θ represents the amount demanded by the arbitrageur in compensation for bearing the risk inherent in the futures position. From Equation 4.29, it is evident that the risk premium depends on Y_o and the distribution of $\tilde{R}$, and will be positive when the rate of change of marginal utility with respect to wealth, $U''(Y_o + E(\tilde{R}))$, is negative. When risk aversion prevails, the measure of absolute risk aversion

$r_A(Y_o + E(\tilde{R})) = -U''(Y_o + E(\tilde{R}))/U'(Y_o + E(\tilde{R}))$, will be positive. Therefore, the size of the risk premium will be a function of the degree of absolute risk aversion, $r_A(Y_o + E(\tilde{R}))$, and the variance of the risky prospect, $\sigma_{\tilde{R}}^2$.

It should now be clear that the total expected return of a risky prospect is comprised of two components: the certainty equivalent of the risky prospect and the risk premium. This was illustrated in Figure 4.4. Therefore, the risk-adjusted expected return may be expressed as:

$$E[\tilde{R}] = R_{ce} + \theta , \qquad (4.30)$$

where R_{ce} represents the certainty equivalent and θ is the risk premium. Rearranging Equation 4.30 and substituting for $E(\tilde{R})$ and θ, the certainty equivalent of the futures position may be expressed as:

$$R_{ce} = \{[100 - ((\frac{FP + c - d}{100})P_m^a + d)] + [(F\tilde{P}_m^e - FP((k)(r)] - [-\frac{1}{2}\frac{U''(Y_o + E(\tilde{R}))}{U'(Y_o + E(\tilde{R}))}\sigma_{\tilde{R}}^2]\}. \qquad (4.31)$$

Equation 4.31 is similar to Equation 4.8 in that it expresses the certain dollar return on the futures position. Unlike Equation 4.8, however, it is more general because it shows explicitly the random component of the margin account and the associated risk premium. For an individual investor, the value of R_{ce} will depend on the cash and futures prices at the time the futures position is initiated, the expectation of interest earned on the margin account, and the subjective risk premium. Over all investors, however, the value of the second term $[(\tilde{FP}_m^e - FP)(k)(r)]$, will equal zero according to the efficient market hypothesis. Even in this case, however, R_{ce} will equal the same constant term expressed in Equation 4.8, and the risk premium will still be positive.

In terms of rates, the certainty equivalent of Equation 4.31 would correspond to a (unobserved) risk adjusted rate of return, r_{ce}^f. By substituting for FP in Equation 4.31 and rearranging, the observed futures rate, r^f, may be expressed in terms of a risk adjusted rate, r_{ce}^f, transactions costs and the risk premium;

$$r^f = \left(\frac{360}{91P_m^a}\right) r_{ce}^f + \left(\frac{360}{91}\right)\left(1 + \frac{c}{100} - \frac{d}{100}\right) + \left(\frac{360}{91P_m^a}\right)(d-100) - \left(\frac{360}{91P_m^a}\right)\left(-\frac{1}{2}\,\frac{U''(Y_o+E(\tilde{R}))}{U'(Y_o+E(\tilde{R}))}\,\sigma_{\tilde{R}}^2\right). \qquad (4.32)$$

Given cash prices (rates), it is clear from Equation 4.32 that the observed rate on a futures contract includes several components; a risk free rate, various transactions costs, and a risk premium. In general terms, the observed rate on a futures contract may be expressed as follows:

$$r^f = \alpha_0 r_{ce}^f + \alpha_1 C + \alpha_2 \theta, \qquad (4.33)$$

where α_0, α_1 and α_2 are undetermined parameters corresponding to the risk free futures rate, r_{ce}^f, transactions costs, C, and the risk premium, θ, respectively.

The forward market

The holding period return of an implicit forward contract may be derived from the yield curve of cash market securities. A yield curve represents the relationship between the yield to maturity and the term to maturity for a group of interest bearing securities with similar characteristics. In particular, it is usually assumed that the credit or default risk of the securities in identical. Thus, yield curves are usually drawn for homogeneous sets of securities such as Treasury bonds, municipals securities, or certain grades of corporate debt. The yield curve is constructed as of a given date and it thus represents the structure of interest rates for a specified group of securities having similar characteristics other than maturity. The yield curve exhibits different shapes according to the current supply and demand for credit, monetary policy, and expectations of future economic conditions.

There exist several hypotheses that purport to explain the term structure of interest rates. One popular theory, called the pure expectations hypothesis of the term structure, has been presented by Hicks (1974) and Lutz (1940). This theory contains two propositions. The first hypothesis states that in competitive equilibrium, for a holding period of any given length, the holding period return on alternative investments will be the same regardless of the maturity of the securities held in the portfolio. Implicit in the mathematics of the first hypothesis is the second proposition which states that under the above circumstances, observed long-term interest rates will equal the (geometric) mean of the current short-term observed rate and short-term rates expected to prevail in the future. When this is true, the long-term investor will earn the same holding period return regardless of whether he holds a long-term security to maturity or invests in successive short-term securities.

Using the same notation as that presented by Malkiel (1966), let ${}_0R_m$ and ${}_0R_n$ represent the currently observed rates on an m-day bill and an n-day bill. In addition, let ${}_mr_{91}$ be the yield on a 91-day bill that is expected to prevail at time m. This rate is not observable in the current period. For a holding period of 0-n, the investor has two options: he can purchase the n-day bill holding it to maturity or, alternatively, he can purchase the m-day bill and reinvest the proceeds at time m at a rate he expects to prevail at time m, i.e., ${}_mr_{91}$. According to the pure expectations hypothesis, competitive equilibrium will ensure that the holding period returns of the two alternatives will be equal. Thus, for the two period case over a holding period of 0-n days, the pure expectations hypothesis may be expressed by the following equation.

$$(1 + \frac{n}{360}\,{}_0R_n) = (1 + \frac{m}{360}\,{}_0R_m)(1 + \frac{91}{360}\,{}_mr_{91}) , \qquad (4.34)$$

where, in accordance with Figure 4.1, the subscript on the left of the observed rates represents the date at which the observation or expectation is made and the subscript on the right represents the maturity of the security in question. In Equation 4.34, which expresses the equality of holding period returns, two points of the cash yield curve, ${}_0R_n$ and ${}_0R_m$, are observed. The 91-day rate expected to prevail at time m, ${}_{m}r_{91}$, is not observed. However, it is implied in the mathematics since Equation 4.33 is one equation in one unknown and can be solved for the value of ${}_{m}r_{91}$ which will equate the holding period returns of the two alternatives. Equation 4.33 can be solved for ${}_{m}r_{91}$ as follows:

$$ {}_{m}r_{91} = \frac{[n/91 \ {}_{0}R_{n} - m/91 \ {}_{0}R_{m}]}{[1 + m/360 \ {}_{0}R_{m}]} . \tag{4.35} $$

Equation 4.35 expresses the forward rate implied in the term structure of cash market securities as of time 0. Suppose that one were to purchase an n-day bill and simultaneously sell an m-day bill. The net effect of these transactions would be an n-m period loan characterized by an implied rate of return given by Equation 4.35.

As an alternative to the spot market transactions, the same effective n-m period loan could be executed by purchasing a futures contract maturing in period m. It has been shown that the important characteristics that distinguish a futures from a foward contract are the cost of transactions, C, and the risk premium, θ. Abstracting from the influence of these characteristics, the rate on a long futures contract establishing an n-m period loan would be identical to the forward rate expressed in Equation 4.35. In other words, the forward rate, ${}_{m}r_{91}$, would equal the certainty equivalent of the futures rate, r_{ce}^{f}, as expressed in Equation 4.33. To be consistent, however, the statistical significance of transactions costs and the risk premium must be determined empirically. If it is found that they are statistically significant, then one would not expect equality between the observed futures rate and the forward rate, even in an efficient market.

The Bias Between Futures and Forward Rates

To the extent that returns on futures and forward transactions represent yields on similar (though not identical) types of investments, market participants will attempt to arbitrage the yields in the futures market and the yields in the spot market (which give rise to implied forward rates) if profitable trading opportunities exist. This potential for arbitrage should result in an equilibrium relationship between observed futures and forward rates such that:

$$r^{f} = {}_{m}r_{91} + C + \theta. \qquad (4.36)$$

Equation 4.36 shows explicitly that the rate on the forward contract, ${}_{m}r_{91}$, is identical to r^{f}_{ce}, the certainty equivalent of the observed futures rate. Taking the difference between the observed futures and the forward rates results in the following:

$$r^{f} - {}_{m}r_{91} = +C + \theta. \qquad (4.37)$$

Equation 4.37 shows that there will be a significant difference between the two rates if transactions costs and the risk premium are statistically significant. Even if arbitrage results in market efficiency, the above condition suggests that the futures rate and foward rate, r^{f} and ${}_{m}r_{91}$, respectively, will not necessarily be identical. The relationship between the two variables is an empirical matter which will depend on the significance of transactions costs and the risk premium. Empirical analysis that has purported to test the efficiency of the futures market by testing for the significance of the difference between r^f and ${}_m r_{91}$ has implicitly assumed that C and θ are equal to zero.

If it is not true that these variables are insignificant, then the nature of the bias between the forward and the futures rate must be explored on the basis of the values of these two terms. Assuming that Treasury securities are used to satisfy the initial margin requirements, then the transactions costs consist only of commissions. Because they represent a fixed cost, the average cost of commission over time will decrease the longer the holding period of the contract. This suggests that in the short run, certainly for day trades or overnight positions, commissions will be much more significant than for longer arbitrage periods where they are spread out over time. Thus, if any bias is imparted due to the existence of commissions, it will be more significant in short holding periods than in long ones.

Finally, if investors who arbitrage the market are risk averse, then they will demand a positive risk premium. Their risk is likely to be positively related to the length of the holding period. Therefore, the risk premium may increase with time to maturity. If, on the other hand, market participants have a preference for risk as speculators, then the risk premium may be negative.

CHAPTER V. EMPIRICAL EVIDENCE ON THE STRUCTURE OF YIELDS

Introduction and Purpose

Chapter IV presented a theoretical analysis of the price structure existing between the futures and the cash markets. In this chapter empirical evidence is presented regarding the major propositions resulting from the theoretical analysis.

The first proposition is that the observed futures rate, r^f, is composed of a risk free rate, r^f_{ce}, transactions costs, C, and a risk premium, θ. The risk premium is compensation for the risk incurred in the margin account. This includes the risk associated with: 1) interest foregone on margin calls which must be paid in cash, and perhaps even more important, 2) the inability to meet margin calls at all. The risk associated with the margin account is not present in the forward market. This distinction between the futures and forward markets has been ignored by previous authors. Thus, one purpose of this chapter is to determine the empirical significance of the risk premium.

In Equation 4.36, the (unobserved) risk free rate on the futures contract was substituted by the forward rate, and the resulting equation is repeated here:

$$r^f = {}_m r_{91} + C + \theta . \qquad (5.1)$$

By rearranging Equation 5.1, the difference between the futures rate and the forward rate may be expressed as:

$$r^{f} - {}_{m}r_{91} = C + \theta . \qquad (5.2)$$

Following from the first proposition, the second states that if transactions costs, C, and the risk premium, θ, are significant to the analysis, then the difference between the futures rate and the forward rate will not be equal to zero, even in an efficient market. If this is true, then tests of the significance of the difference from zero are not appropriate tests for the efficiency of the market.

Specific hypothesis

The difference between the futures and forward rates is analyzed using summary statistics. The relationship between the difference and transaction costs and the risk premium is explored econometrically. Three hypotheses are tested. The first null hypothesis is that the difference between the two sets of rates is equal to zero:

$$H_{o}: \ D = 0, \quad H_{A}: \ D \neq 0 \ , \qquad (5.3)$$

where $D = r^{f} - {}_{m}r_{91}$. Tests of the significance of the difference are conducted by contract, by the average of all contracts, and over time, by quarters preceding the maturity of the futures contract.

The second and third hypotheses concern the significance of transactions costs and the risk premium. The null hypothesis of both tests states that the coefficient on each of these variables used in a regression analysis to explain D is equal to zero. The alternative hypothesis states that they are not equal to zero:

$$H_o\text{: } B_C = 0, \quad H_A\text{: } B_C \neq 0 , \tag{5.4}$$

where B_C is the coefficient on C, the variable representing transactions costs, and:

$$H_o\text{: } B_\theta = 0, \quad H_A\text{: } B_\theta \neq 0 , \tag{5.5a}$$

where B_θ is the coefficient on θ, the measure of risk.

Measurement of risk and the risk premium

It is clear from Equation 4.29 that the risk premium is a function of the subjective utility function and the variance of the return on the risky prospect. Difficulties arise, however, in the empirical estimation of the risk premium. Although the subjective utility function may be a very useful concept in theoretical analysis, the measurement of its parameters may be impossible. Similarly, the measurement of the variance of the returns on the futures position is equally difficult to obtain ex ante to the initiation of the position.

Given the difficulty of estimating various economic concepts, proxies are often used with the hope that they capture the essence of the variable in question. With respect to the risk premium, two separate proxies were used to simulate the risk associated with the variance in the return on the futures position. The first proxy used was a variable called days to maturity (DTM). The second was a time-series variance of the futures rate. In the regression analyses which follow, the coefficients on each of these variables were tested for their significance. Coefficients significantly different from zero imply that commissions and a risk premium are significant components of the observed futures price.

Data

Futures data were obtained from the International Monetary Market of the Chicago Mercantile Exchange. The settlement prices for each of the 14 contracts analyzed were gathered on a daily basis over the nine months preceding the maturity date of each contract. Approximately 180 observations of the IMM index were recorded on each contract. A list of the futures contracts considered in the analysis is shown in the first column of Table A.1 of Appendix A. The first two contracts traded on the Treasury bill futures market (March 1976 and June 1976) were not considered in this analysis because each of them traded for less than 9 months before they expired.

The cash data, used to calculate forward rates implied in the term structure, were gathered from the "Composite Closing Quotations for U.S. Government Securities", provided by the Federal Reserve Bank of New York. Rates on 84 different Treasury bill issues were used in the calculation of the forward rates. The bid and ask yields were recorded for each day that a Treasury bill was used. Table A.1 of Appendix A lists the bills that were used in the calculation of forward rates and the number of observations (bid and asked yields) of each bill used. The last column of Table A.1 shows the total number of yield observations on the cash bills which correspond to the relevant futures contract.

The total data set consisted of observations of the IMM index and corresponding observations of bid and ask yields of the Treasury bills used to calculate forward rates. There were various holidays and other days when either the cash market or the futures market was closed. All observations for these days were discarded. In other words, futures and cash data were used only on those days when both exchanges were open. The total data set, including the daily price data for the futures contracts and the corresponding yield data on the relevant Treasury bills, consisted of 2,463 observations. From this data set, futures and forward rates were constructed and analyzed by contract and over time.

Data Manipulation

The initial objective of the analysis was to compare the futures rate to the forward rate on a day by day basis over the nine month holding period previous to the maturity date of each of the 14 futures contracts. To analyze the difference, D, of the two rates over time, the raw data were manipulated to calculate a daily futures rate and the corresponding forward rate.

Futures rates

The futures data were transformed such that continuously compounded yields replaced the observations of the IMM index. First, the annual discount rate on the futures contract was found by subtracting the IMM index from 100:

$$Y^F = 100 - \text{IMM index} , \qquad (5.5b)$$

where Y^F is the annual discount rate on the futures contract. Using the standard formula for pricing Treasury bills, the futures discount rate was used to find the price of the futures contract:

$$FP = [100 - (\frac{91}{360})Y^F(100)] . \qquad (5.6)$$

From investment theory it is well-known that the future value is related to present value over time by the rate of interest. As a natural exponential function, this may be expressed as follows:

$$F = Pe^{rt} , \qquad (5.7)$$

where F is the future value, P is present value, r is the continuously compounded rate of return, and t is the holding period. Rearranging Equation 5.7, the continuously compounded rate of return may be solved for as follows:

$$r = (\frac{1}{t})\ln(\frac{F}{P}) , \qquad (5.8)$$

where ln is the natural logarithm.

At their maturity, the bills delivered on the futures contract have a value of $100.00, which is comparable to F in Equation 5.8. By substituting the appropriate futures variables for the holding period, t, and the present value, P, in Equation 5.8, the continuously compounded rate of return on a futures contract was computed as follows:

$$r^f = [(\frac{365}{91})\ln(\frac{100}{FP})]. \quad (5.9)$$

In the manner of Equation 5.9, continuously compounded futures rates were calculated for each futures contract on each day of the nine months preceding the maturity date of the contract.

Forward rates

Due to various measurement problems, the cash data was manipulated prior to the calculation of the forward rates.

Maturity of cash bills and futures contracts Treasury bills, it will be recalled, are issued in maturities of 91 days, 182 days, and one year. The futures contract specifies delivery of the 90-day bill. Of the outstanding Treasury bills, there is only one--the 182-day bill issued three months prior to the maturity date of the futures contract--which corresponds exactly to the delivery instrument specified by the contract. At delivery, the short futures position has two options: deliver the 91-day bill issued the day before expiration of the futures contract or deliver the outstanding six-month bill which has 91 days to maturity. The delivery instruments are identical. This means that during the three months prior to the maturity date of the futures contract, time m in Figure 4.1, there exist cash bills which coincide perfectly to the time dimension of the delivery vehicle. For this three month period, the means of the bid-ask discount rate on Treasury bills maturity at time m and n, respectively, were calculated as follows:

$$Y_M = (Y_{MB} + Y_{MA})/2 , \qquad (5.10a)$$

$$Y_N = (Y_{NB} + Y_{NA})/2 , \qquad (5.10b)$$

where Y_M and Y_N are the discount rates on bills which mature at time m and n, respectively, Y_{MB} and Y_{NB} are the bid yields, and Y_{MA} and Y_{NA} are the ask yields.

The delivery issue on a futures contract does not exist prior to three months before the contract's expiration date. Therefore, the yield curve of cash bills was interpolated to estimate what the yield would have been on hypothetical bills with the same time dimensions relevant to the futures contract. For example, consider the 12-20-79 contract, which called for delivery of a 91-day bill maturing on 3-19-80. Prior to 9-20-79, the delivery instrument on the futures contract had not yet been issued. Therefore, to calculate a yield comparable to a bill maturing on 3-19-80, the yield curve had to be interpolated using one-year bills, one maturing just prior to 3-19-80, and one maturing just after 3-19-80. The one-year bills used were 3-4-80 and 4-1-80.

As with the perfectly matched bills which exist during the three months prior to the maturity of the futures contract, the yields on the hypothetical bills also represented the mean of the bid-ask yields.

They were calculated as follows:

$$Y_{ML} = (Y_{MBL} + Y_{MAL})/2 \tag{5.11a}$$

$$Y_{MU} = (Y_{MBU} + Y_{MAU})/2 \tag{5.11b}$$

$$Y_{NL} = (Y_{NBL} + Y_{NAL})/2 \tag{5.11c}$$

$$Y_{NU} = (Y_{NBU} + Y_{NAU})/2\ , \tag{5.11d}$$

where Y_{ML} is the mean of the bid-ask yields of the one-year bill maturing just prior to the hypothetical bill which would mature at time m. The bid yield is Y_{MBL} and the ask yield is Y_{MAL}. Calculated in a similar manner, Y_{MU} is the mean yield of the one-year bill maturing just after the time m hypothetical bill. Y_{NL} and Y_{MU} are the mean yields on the one-year bills which surround the hypothetical bill maturing at time n.

The yields represented by Equations 5.11a through 5.11d were used to estimate the yields of the hypothetical bills maturing at times m and n. It was assumed that the yield curve could be linearly approximated and the general form for the linear interpolation of Y_M was:

$$Y_M = \frac{[(D^u - D^*)(Y_{ML}) + (D^* - D^L)(Y_{MU})]}{(D^u - D^L)}\ , \tag{5.12}$$

where $(D^u - D^*)$ is the number of days between the maturity date of the hypothetical bill and the one-year bill maturing just after it, $(D^* - D^L)$ is the number of days between the maturity date of the hypothetical bill and the one-year bill maturing just prior to it, and (D^u-D^L) is the number of days between the maturity dates of the one-year bills used in the interpolation. The linear interpolation of Y_N was calculated similarly. Figure A.1 in Appendix A illustrates the interpolation process and Table A.2, also in Appendix A, lists the equations used to interpolate the cash bills corresponding to each of the futures contracts.

Averaging the bid-ask yields of all cash bills and interpolating the one-year bills used during the time period prior to three months before the maturity date of the futures contract (when perfectly matched bills do not exist) results in a time series of Treasury bill discount rates on bills which correspond to the maturity date of the futures contract, time m, and the maturity date of the delivery instrument, time n. There were 14 separate time series of Treasury bill discount rates, Y_M and Y_N, one series for each futures contract, and the discount rates were then used to construct forward rates implied from the term structure.

Continuously compounded forward rates To make the forward rates consistent with the futures rates, they were constructed on the basis of continuously compounded rates of return. On a per dollar basis and a 365 day year, the price of a 91-day Treasury bill due to mature n periods (one period equals 91 days) in the future is:

$$P_{n,t} = e^{-[R_{t+1,t}(\frac{91}{365}) + r_{t+2,t}(\frac{91}{365}) + \ldots + r_{t+n,t}(\frac{91}{365})]} \quad (5.13)$$

where $P_{n,t}$ is the current price of a Treasury bill maturing in period t + n, $R_{t+1,t}$ is the currently observed spot rate on a 91-day Treasury bill due to mature in period t+1, and r_{t+i}, i = 2...n, are the one period continuously compounded forward rates expected to prevail in future periods. Equation 5.13 may be rewritten as:

$$P_{n,t} = P_{n-1,t}\, e^{-r_{t+n,t}\left(\frac{91}{365}\right)} \quad (5.14)$$

and the continuously compounded forward rate may be solved for directly:

$$r_{t+n,t} = \left(\frac{365}{91}\right) \ln\left(\frac{P_{n-1,t}}{P_{n,t}}\right). \quad (5.15)$$

To calculate a forward rate comparable to the futures rate, it was necessary to derive the cash prices for Treasury bills having the same maturity as that of the futures contract, time m, and of the delivery instrument, time n. Using the Treasury bill discount rates described previously, the prices of the m- and n-day cash Treasury bills were calculated using the standard formula for the price of a Treasury bill:

$$P_M = [100 - \left(\frac{DTM}{360}\right) Y_M (100)] \quad (5.16a)$$

and

$$P_N = [100 - \left(\frac{DTM + 91}{360}\right) Y_N (100)]\,, \quad (5.16b)$$

where DTM represents days to maturity.

Using these prices in Equation 5.15 the continuously compounded forward rates were computed as follows:

$$_m r_{91} = \left(\frac{365}{91}\right) \ln(P_M/P_N). \quad (5.17)$$

Corresponding to the futures rates, forward rates were calculated on a daily basis for the nine months immediately preceding the maturity date of each futures contract. To have expanded the time series beyond nine months would have necessitated using coupon instead of discount securities with the result that the forward rates would not have been directly comparable to the futures rates.

Having calculated the futures and the forward rates, the difference, D, between them was computed:

$$D = r^{f} - {}_{m}r_{91}. \qquad (5.18)$$

With these calculations completed, the basic data set consisted of observations on the futures rate, f^{r}, the forward rate, ${}_{m}r_{91}$, their difference, D, and days to maturity, DTM, of each contract.

Summary Statistics and Analysis

Having calculated the futures and forward rates, the difference between them and the days to maturity for each observation by contract, summary statistics of the difference between the futures and the forward rate were calculated by contract and over time. The major objective of the analysis of summary statistics was to determine if the difference, D, was statistically different from zero. This part of the analysis abstracts from transactions costs and any risk premiums, only the difference between the two rates is considered.

Table 5.1 shows summary statistics on the difference between the

Table 5.1. Summary statistics on the difference between the futures rate and the forward rate for each contract and over all contracts, expressed over time and in nonannualized basis points

		12-20-79	9-20-79	6-21-79	3-22-79	12-21-78	9-21-78	6-22-78
Quarter Nearest Maturity	U[a]	-39.17	-18.31	-8.70	-9.71	-25.78	-21.38	-14.45
		(-10.05)[b]	(-6.32)	(-6.36)	(-8.38)	(-6.54)	(-8.29)	(-8.45)
	|U|[c]	40.89	21.62	11.15	10.46	32.28	24.04	16.49
		(11.42)	(9.07)	(10.88)	(10.06)	(10.75)	(11.20)	(12.11)
	N[d]	59	60	60	59	59	60	60
Second Quarter	U	-90.04	-60.14	-93.84	-71.19	-86.51	-61.88	-44.02
		(-23.84)	(-23.31)	(-21.48)	(-24.68)	(-29.76)	(-22.85)	(-32.27)
	|U|	90.04	60.14	93.84	71.19	86.51	61.88	44.02
		(23.84)	(23.31)	(21.48)	(24.68)	(29.76)	(22.85)	(32.27)
	N	62	60	60	60	62	62	59
Third Quarter	U	-108.80	-81.71	-84.10	-88.44	-55.26	-18.26	-35.24
		(-18.51)	(-62.30)	(-19.66)	(-33.74)	(-17.96)	(-13.77)	(-47.86)
	|U|	108.80	81.71	84.10	88.44	55.26	18.26	35.24
		(18.51)	(62.30)	(19.66)	(33.74)	(17.96)	(13.77)	(47.86)
	N	54	53	48	63	58	53	49
Entire Nine Months	U	-78.68	-52.41	-60.65	-57.23	-56.37	-34.78	-30.90
		(-23.08)	(-21.80)	(-16.71)	(-20.10)	(-21.07)	(-17.04)	(-24.32)
	|U|	79.26	53.39	61.52	57.47	58.51	35.69	31.63
		(23.79)	(23.45)	(17.37)	(20.38)	(24.35)	(18.31)	(27.30)
	N	175	173	168	182	179	175	168

[a] Mean.

[b] t statistics for testing the null hypothesis that the mean equals zero.

[c] Mean absolute value.

[d] Number of observations.

Table 5.1. (Cont.)

3-23-78	12-22-77	9-22-77	6-23-77	3-24-77	12-23-76	9-23-76	Over all Contracts
-16.64	-22.64	-6.78	-6.99	-2.62	-9.32	-3.12	-14.68
(-14.62)	(-16.10)	(-7.87)	(-6.53)	(-3.12)	(-9.50)	(-5.48)	(-13.97)
16.72	22.64	7.79	8.92	5.11	10.17	4.18	16.55
(14.96)	(16.10)	(11.10)	(11.21)	(8.31)	(12.35)	(9.50)	(18.34)
58	59	60	61	60	59	61	63
-38.80	-33.66	-11.76	-9.48	-20.65	-22.80	-4.28	-46.27
(-20.77)	(-16.30)	(-6.62)	(-5.12)	(-15.59)	(-17.62)	(-2.44)	(-64.96)
38.80	33.66	16.31	14.45	20.79	22.69	12.05	47.57
(20.27)	(16.30)	(15.82)	(11.90)	(16.16)	(18.38)	(12.11)	(71.04)
60	61	62	62	60	62	62	64
4.43	7.89	48.76	-11.81	11.16	16.97	-18.76	-27.87
(2.39)	(2.91)	(13.97)	(-5.13)	(4.11)	(8.99)	(-4.31)	(-14.62)
12.68	18.23	48.76	16.44	18.90	18.19	26.54	42.96
(12.92)	(11.14)	(13.97)	(10.34)	(10.20)	(10.82)	(7.52)	(33.99)
58	59	56	48	60	60	55	73
-17.25	-16.33	8.96	-9.24	-4.03	-5.23	-8.36	-30.01
(-10.46)	(-9.09)	(3.72)	(-9.14)	(-2.83)	(-3.52)	(-5.32)	(-37.80)
22.91	24.94	23.65	13.04	14.93	17.21	13.83	36.19
(19.23)	(22.51)	(13.48)	(17.97)	(15.98)	(20.37)	(10.36)	(53.13)
176	179	178	171	180	181	173	2463

futures and the forward rates for each contract and over all contracts, expressed over time and in nonannualized basis points. Each column in Table 5.1 represents a separate futures contract with the most recent contract on the left and the average over all contracts on the right. The first three rows represent analysis by quarters preceding the maturity date of each contract and the bottom row includes summary statistics over the entire nine months of trading. The mean of the difference of the futures and forward rates for each quarter preceding maturity is shown above the t-statistic (in parentheses) which tests its significance from zero. All t-values are significant at the 5 percent level. The number of observations included in each of the averages is also shown.

For the first quarter of each contract, the statistics show that the mean of the difference is negative. Averaged over all contracts, the forward rate exceeds the futures rate by approximately 15 basis points. Because the average of deviations of opposite sign tend to offset one another, the mean of the absolute value of the basis point differential was also calculated. The mean absolute value, signified by $|U|$, is larger than the arithmetic mean of the difference, indicating that for each contract, there were positive observations of the difference, that is, the futures rate exceeded the forward rate. All t-statistics, shown in parentheses below the mean absolute difference, are significant.

In the second quarter preceding the maturity of each contract, the forward rate increased relative to the futures rate. In Table 5.1, this is reflected in larger values for the mean difference. For most of the contracts, the mean absolute difference is identical to the mean difference indicating that there were very few, if any, observations in the second quarter where the futures rate exceeded the forward rate. The growing divergence between the futures and forward rates in the second quarter is also reflected by very large t-statistics. Over all contracts, the forward rate exceeded the futures rate by an average of 46 basis points.

During the third quarter preceding maturity, the mean difference declined for most contracts as the futures rate rose relative to the forward rate. In fact, for five of the contracts, the mean difference became positive with futures rates exceeding forward rates. Over all contracts, the forward rate was an average of 28 basis points greater than the futures rate. However, the large mean absolute value suggests that there were a large number of observations where the mean difference between futures and forward was positive.

Statistics over the entire nine months of trading show that with the exception of one contract, the mean difference was negative. Over all contracts, the forward rate exceeded the futures rate by 30 basis points.

The statistics in Table 5.1 clearly indicate that the difference between the futures and forward rates, both over time and by contract, is significantly different from zero. On the average, the difference is small at the time of maturity, increases during the first and second quarters, and then begins to decrease during the third quarter. The difference also increases across contracts. In the second quarter for example, the 9-23-76 contract had an average difference of only -4 basis points. However, by the time the 12-20-79 contract was trading, the difference was -90 basis points. This increasing discrepancy between the futures and forward rates over contracts may suggest that opportunities for arbitrage have increased rather than decreased as the market has matured. The opposite was expected ex ante to the analysis.

The statistics in Table 5.1 are presented on a nonannualized basis. If the basis point differential is any indication of potential arbitrage opportunities, then the results in Table 5.1 suggest that the greatest opportunity for arbitrage would occur during the second quarter preceding the maturity date of the futures contract. However, it must be recognized that for a hold-to-maturity strategy, arbitrage profits realized in the first quarter can be earned in a shorter period of time than those in the second or third quarters. For this reason, the basis point differentials were adjusted to reflect the holding period of the contract. The following adjustment was used to annualize the basis point differential:

$$D^{A} = D\left(\frac{365}{DTM}\right) , \qquad (5.19)$$

where D^{A} is the annualized basis point differential and DTM, as before, represents the days to maturity of the contract.

The results of the annualized analysis are presented in Table 5.2.

Table 5.2. Summary statistics on the difference between the futures rate and the forward rate for each contract and over all contracts, expressed over time and in annualized basis points

		12-20-79	9-20-79	6-21-79	3-22-79	12-21-78	9-21-78	6-22-78
Quarter Nearest Maturity	U[a]	-429.21	-79.44	4.29	-72.02	-140.82	-102.16	-82.13
		(-6.87)[b]	(-3.50)	(0.12)	(-5.83)	(-4.24)	(-4.19)	(-4.44)
	|U|[c]	499.72	163.93	151.37	100.26	245.64	193.50	145.11
		(9.48)	(12.75)	(4.82)	(12.11)	(12.20)	(16.57)	(14.54)
	N[d]	59	60	60	59	59	60	60
Second Quarter	U	-241.43	-173.99	-248.93	-204.56	-233.75	-181.38	-128.31
		(-40.48)	(-15.55)	(-34.67)	(-20.42)	(-36.45)	(-17.26)	(-20.31)
	|U|	241.43	173.99	248.93	204.56	233.75	181.38	128.31
		(40.48)	(15.55)	(34.67)	(20.42)	(36.45)	(17.26)	(20.31)
	N	62	60	60	60	62	62	59
Third Quarter	U	-189.43	-143.72	-148.28	-145.91	-96.16	-32.10	-60.66
		(-15.63)	(-59.38)	(-16.57)	(-30.49)	(-14.55)	(-12.07)	(-32.03)
	|U|	189.43	136.72	148.28	145.91	96.16	32.10	60.66
		(15.63)	(59.33)	(16.57)	(30.49)	(14.55)	(12.07)	(32.03)
	N	54	53	48	63	58	53	49
Entire Nine Months	U	-288.69	-129.73	-129.73	-141.29	-158.54	-109.01	-92.09
		(-12.69)	(-14.02)	(-8.12)	(-21.00)	(-13.09)	(-10.67)	(-12.64)
	|U|	312.47	159.03	185.33	150.45	192.09	140.32	114.58
		(14.97)	(26.35)	(15.10)	(27.24)	(21.81)	(18.18)	(22.83)
	N	175	178	168	182	179	175	168

[a]Mean.

[b]t statistics for testing the null hypothesis that the mean equals zero.

[c]Mean absolute value.

[d]Number of observations.

Table 5.2. (Cont.)

3-23-78	12-22-77	9-22-77	6-23-77	3-24-77	12-23-76	9-23-76	Over all Contracts
-142.85	-197.54	-48.79	-77.78	-31.72	-72.64	-33.69	-107.81
(-16.21)	(-24.77)	(-3.12)	(-7.43)	(-3.97)	(-6.23)	(-5.41)	(-13.68)
146.43	197.54	86.68	88.83	50.60	95.29	44.28	157.16
(18.99)	(24.77)	(6.90)	(10.00)	(8.26)	(11.37)	(8.85)	(20.66)
58	59	60	61	60	59	61	63
-115.05	-100.53	-38.24	-26.93	-58.21	-67.47	-17.22	-130.66
(-16.00)	(-13.11)	(-7.10)	(-5.53)	(-14.67)	(-14.47)	(-3.20)	(-32.12)
115.05	100.53	48.29	39.70	58.60	67.79	36.50	133.81
(16.00)	(13.11)	(12.58)	(12.71)	(15.15)	(14.78)	(10.56)	(35.22)
60	61	62	62	60	62	62	64
5.27	10.41	78.97	-21.37	15.15	26.80	-26.94	-47.12
(1.73)	(2.37)	(15.06)	(-5.20)	(3.65)	(8.78)	(-4.05)	(-14.01)
20.70	29.72	78.97	28.85	29.25	29.24	41.15	70.68
(13.70)	(12.26)	(15.06)	(9.68)	(11.35)	(11.06)	(8.03)	(29.37)
58	59	56	48	60	60	55	73
-84.56	-95.94	-4.92	-43.51	-24.93	-37.91	-25.87	-97.33
(-13.60)	(-12.85)	(-0.68)	(-9.27)	(-6.27)	(-7.00)	(-7.34)	(-32.09)
94.30	109.17	70.88	54.18	46.15	63.97	40.60	123.52
(17.59)	(17.15)	(14.54)	(13.58)	(17.02)	(16.74)	(15.48)	(47.20)
176	179	178	171	180	181	178	2463

They are consistent with those reported in Table 5.1 in that the difference between the futures and forward rates is significantly different from zero, both by contract and over time. However, due to the annualization, the differentials have increased substantially for each contract and over all contracts. For example, the differential of the first quarter of the most recent contract considered, 12-20-79, was over four hundred basis points. This is a substantial difference which would seem to indicate strong opportunities for arbitrage. However, the annualization process assumes that the differentials that occur in the short run will be replicated throughout the year, perhaps an assumption which is too strong in such a volatile market.

To illustrate the summary statistics included in Tables 5.1 and 5.2, the difference between the futures rate and the forward rate was plotted by days to maturity for each contract and over all contracts. Figure 5.1 shows the average difference over all 14 contracts plotted by days to maturity. The difference is expressed in basis points. Points below the dashed line in Figure 5.1 indicate that the futures rate is less than the forward rate and points above the line indicate the opposite. Figure 5.1 clearly shows that the futures and forward rates coincide at the maturity date of the futures contract. However, as days to maturity increase, the forward rate rises relative to the futures rate and overall, the difference seems to follow a quadratic pattern as the days to maturity increase. At 90 days to maturity, there is a discontinuity in the quadratic curve caused by the change

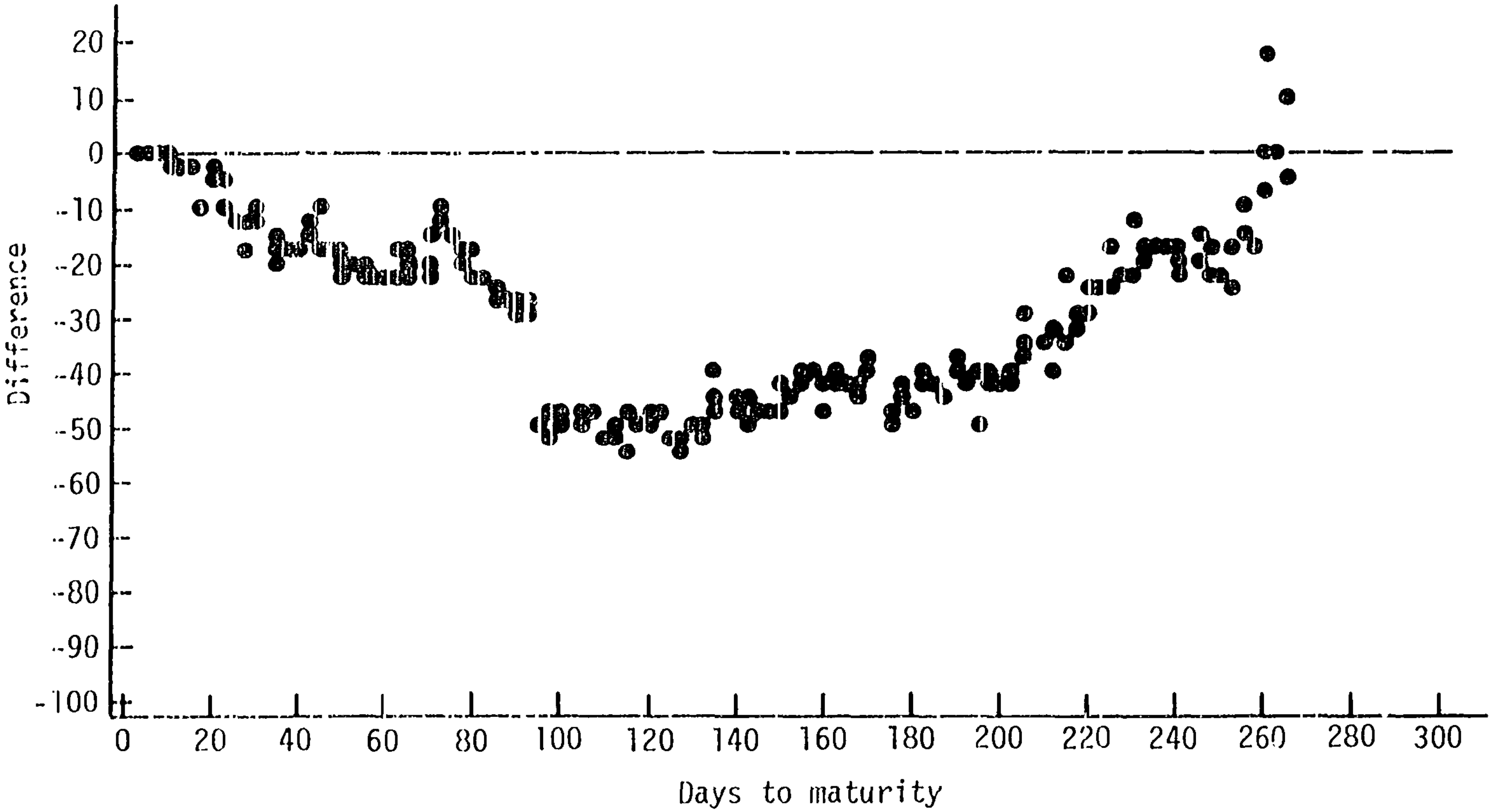

Figure 5.1. Plot of futures rate minus forward rate by days to maturity over all contracts, expressed in basis points

in the procedure for calculating forward rates. Prior to 90 days, Treasury bills with the exact maturity dimensions of the futures contract are used to calculate forward rates. Beyond 90 days, one-year bills are used in the interpolation process previously described to calculate the forward rates. It is apparent that the interpolation of the one-year bills and the creation of hypothetical bills which have the exact time dimensions of the futures contract and its delivery issue imparts an upward bias to the forward rates. Thus, the basis point differentials in the second and third quarters may not be as large as this analysis suggests. Graphs similar to Figure 5.1 were plotted for each of the 14 contracts and are included in Appendix B.

To further illustrate the pattern of futures and forward rates, the difference and the absolute difference were averaged and listed by weeks to maturity. Table 5.3 shows the average deviation (D) and average absolute deviation ($|D|$) of the futures minus the forward rate, expressed in basis points for each contract and over all contracts, by weeks to maturity. As with Figure 5.1, Table 5.3 shows that the futures and forward rates have similar values at the maturity date of the contract, but diverge as weeks to maturity increase. The average absolute difference gives some indication of the degree of variation of the difference near the maturity date of the futures contract. Three to five weeks prior to maturity, however, the average absolute deviation becomes very similar (in absolute value) to the average deviation indicating that very few of the observations on the difference are positive.

Table 5.3. Average deviation (D) and average absolute deviation (|D|) of the futures rate from the forward rate, expressed in basis points for each contract and over all contracts.

Weeks to Maturity	12-20-79 D	12-20-79 |D|	9-20-79 D	9-20-79 |D|	6-21-79 D	6-21-79 |D|
1	- 5.27	10.38	- 0.53	2.26	10.35	10.35
2	- 27.04	27.04	5.77	5.77	3.02	5.94
3	- 41.83	41.83	11.43	11.43	- 2.16	5.26
4	- 59.68	59.68	- 1.76	4.05	- 10.73	10.73
5	- 69.76	69.76	- 4.58	4.82	- 3.64	4.19
6	- 43.65	43.65	- 12.23	12.23	- 9.24	9.24
7	- 61.89	61.89	- 22.28	22.28	- 7.29	7.29
8	- 68.80	68.80	- 23.04	23.04	- 11.96	12.11
9	- 15.89	17.58	- 25.44	25.44	- 25.16	25.16
10	- 9.51	14.24	- 34.39	34.39	- 22.97	22.97
11	1.52	10.72	- 28.42	28.42	- 9.71	9.71
12	- 36.84	36.84	- 43.70	43.70	- 10.43	10.43
13	- 71.89	71.89	- 66.72	66.72	- 13.55	13.55
14	- 68.04	68.04	- 76.84	76.84	- 42.92	42.92
15	- 61.90	61.90	- 95.41	95.41	- 64.40	64.40
16	- 75.15	75.15	- 65.32	65.32	- 77.54	77.54
17	- 62.76	62.76	- 79.81	79.81	- 75.68	75.68
18	- 68.29	68.29	- 36.50	36.50	- 78.75	78.75
19	- 67.98	67.98	- 54.23	54.23	- 90.73	90.73
20	- 98.68	98.68	- 37.25	37.25	- 86.12	86.12
21	-108.77	108.77	- 50.44	50.44	- 94.58	94.58
22	- 83.86	83.86	- 62.87	62.87	-106.52	106.52
23	- 96.49	96.49	- 47.72	47.72	-117.78	117.78
24	-110.62	110.62	- 51.14	51.14	-139.51	139.51
25	-143.48	143.48	- 60.72	60.72	-155.17	155.17
26	-155.55	155.55	- 70.38	70.38	-136.67	136.67
27	-136.30	136.30	- 67.13	67.13	-102.94	102.94
28	-162.60	162.60	- 77.96	77.96	-107.82	107.82
29	-158.91	158.91	- 85.16	85.16	-108.72	108.72
30	-144.38	144.38	- 86.74	86.74	-100.77	100.77
31	-116.90	116.90	- 77.70	77.70	-106.22	106.22
32	-112.11	112.11	- 79.21	79.21	- 86.29	86.29
33	-116.46	116.46	- 92.19	92.19	- 76.68	76.68
34	- 81.82	81.82	- 88.15	88.15	- 54.10	54.10
35	- 61.07	61.07	- 86.21	86.21	- 32.87	32.87
36	- 39.55	39.55	- 90.77	90.77	- 45.01	45.01
37	- 54.72	54.72	- 71.21	71.21	.	.
38	.	.				
39	.	.				

Table 5.3. (Cont.)

Weeks to Maturity	3-22-79		12-29-78		9-21-78	
	D	|D|	D	|D|	D	|D|
1	1.34	1.58	2.49	3.01	4.13	4.13
2	0.54	3.64	- 3.01	3.10	7.33	7.33
3	- 4.72	4.72	1.49	5.42	6.87	7.22
4	- 10.09	10.09	17.02	17.02	- 13.89	14.46
5	- 15.57	15.57	10.96	29.19	- 20.58	20.58
6	- 11.48	11.48	- 53.91	53.91	- 4.14	4.60
7	- 16.77	16.77	- 33.25	33.25	- 13.16	13.16
8	- 17.41	17.41	- 10.71	15.48	- 37.13	37.13
9	- 17.20	17.20	- 45.73	45.73	- 42.90	42.90
10	- 5.43	7.21	- 66.69	66.69	- 36.38	36.38
11	- 1.34	1.91	- 50.73	50.73	- 28.64	28.64
12	- 6.29	6.29	- 52.65	52.65	- 42.08	42.08
13	- 32.29	32.29	- 32.05	32.05	- 52.44	52.44
14	- 66.24	66.24	- 41.40	41.40	- 70.28	70.28
15	- 75.03	75.03	- 62.65	62.65	- 74.53	74.53
16	- 76.89	76.89	- 67.96	67.96	- 74.13	74.13
17	- 67.95	67.95	- 81.08	81.08	- 79.52	79.52
18	- 90.55	90.55	- 90.82	90.82	- 93.30	93.30
19	-106.66	106.66	-104.44	104.44	- 86.20	86.20
20	- 84.79	84.79	-107.89	107.89	- 72.72	72.72
21	- 95.07	95.07	-112.59	112.59	- 63.70	63.70
22	- 74.00	74.00	- 95.35	95.35	- 52.40	52.40
23	- 52.65	52.65	- 87.09	87.09	- 39.71	39.71
24	- 45.00	45.00	- 88.40	88.40	- 27.49	27.49
25	- 34.73	34.73	- 99.96	- 99.96	- 30.68	30.68
26	- 48.01	48.01	-100.71	100.71	- 37.90	37.90
27	- 68.29	68.29	- 91.96	91.96	- 32.22	32.22
28	- 84.55	84.55	- 81.73	81.73	- 25.72	25.72
29	- 97.87	97.87	- 73.77	73.77	- 26.78	26.78
30	-106.09	106.09	- 65.51	65.51	- 24.39	24.39
31	-103.50	103.50	- 67.39	67.39	- 15.56	15.56
32	-110.79	110.79	- 59.68	59.68	- 12.22	12.22
33	-107.98	107.98	- 42.82	42.82	- 18.34	18.34
34	-103.26	103.26	- 33.79	33.79	- 12.24	12.24
35	- 98.18	98.18	- 36.52	36.52	- 9.32	9.32
36	- 87.06	87.06	- 37.40	37.40	- 12.68	12.68
37	- 74.57	74.57	- 26.29	26.29	- 4.32	4.32
38	- 55.73	55.73	- 22.28	22.28	.	.
39	- 46.30	46.30	.	.	.	.

Table 5.3. (Cont.)

Weeks to Maturity	6-22-78 D	6-22-78 |D|	3-23-78 D	3-23-78 |D|	12-22-77 D	12-22-77 |D|
1	1.16	2.92	- 1.43	1.43	- 1.86	1.86
2	3.58	3.58	- 3.24	4.16	- 5.03	5.03
3	5.05	5.05	- 7.29	7.29	- 13.15	13.15
4	1.73	4.89	- 13.72	13.72	- 19.87	19.87
5	- 17.12	17.12	- 16.65	16.65	- 18.68	18.68
6	- 24.41	24.41	- 17.60	17.60	- 25.73	25.73
7	- 15.34	15.34	- 18.66	18.66	- 27.04	27.04
8	- 23.34	23.34	- 25.68	25.68	- 30.66	30.66
9	- 36.18	36.18	- 21.05	21.05	- 34.10	34.10
10	- 27.44	27.44	- 24.75	24.75	- 33.91	33.91
11	- 17.08	17.08	- 18.10	18.10	- 31.84	31.84
12	- 14.22	14.22	- 23.55	23.55	- 30.87	30.87
13	- 17.85	17.85	- 30.06	30.06	- 19.94	19.94
14	- 43.48	43.48	- 45.58	45.58	- 30.19	30.19
15	- 55.44	55.44	- 41.89	41.89	- 49.24	49.24
16	- 58.38	58.38	- 50.57	50.57	- 63.27	63.27
17	- 57.41	57.41	- 58.40	58.40	- 58.18	58.18
18	- 47.27	47.27	- 54.84	54.84	- 41.92	41.92
19	- 41.91	41.91	- 54.95	54.95	- 40.43	40.43
20	- 40.73	40.73	- 39.68	39.68	- 32.21	32.21
21	- 39.59	39.59	- 38.73	38.73	- 24.91	24.91
22	- 39.14	39.14	- 39.22	39.22	- 24.84	24.84
23	- 42.60	42.60	- 25.87	25.87	- 10.89	10.89
24	- 40.54	40.54	- 15.04	15.04	- 15.79	15.79
25	- 29.31	29.31	- 17.74	17.74	- 19.05	19.05
26	- 32.40	32.40	- 13.85	13.85	- 21.11	21.11
27	- 42.99	42.99	- 10.82	10.82	- 16.20	16.20
28	- 31.28	31.28	- 10.15	10.15	- 14.94	14.94
29	- 38.38	38.38	- 13.91	13.91	- 11.43	11.43
30	- 40.45	40.45	- 9.07	9.07	- 8.02	8.02
31	- 32.94	32.94	1.27	5.15	10.66	11.67
32	- 30.90	30.90	4.52	6.14	17.72	17.72
33	- 35.92	35.92	17.79	17.79	26.22	26.22
34	- 33.78	33.78	16.97	16.97	37.05	37.05
35	- 31.45	31.45	10.91	10.91	18.87	18.87
36	- 32.51	32.51	27.64	27.64	1.86	6.14
37	.	.	19.01	19.01	3.81	9.64
38	.	.	6.53	6.53	42.16	42.16
39	.	.	.	.	.	.

Table 5.3. (Cont.)

Weeks to Maturity	9-22-77		6-23-77		3-24-77	
	D	|D|	D	|D|	D	|D|
1	1.61	2.65	- 1.57	1.57	- 1.91	2.15
2	- 1.26	2.10	- 0.23	1.26	- 0.81	1.97
3	0.01	2.71	- 4.45	4.45	- 0.67	1.55
4	- 10.21	10.77	- 9.01	9.01	- 5.12	5.12
5	- 10.95	10.95	- 11.59	11.59	- 1.73	2.49
6	- 8.20	8.20	- 21.92	21.92	- 4.92	4.96
7	1.70	3.00	- 13.09	13.09	- 3.56	3.56
8	- 10.49	10.49	- 7.02	7.02	- 4.30	4.30
9	- 12.57	12.57	- 12.18	12.18	4.77	4.77
10	- 6.87	6.87	- 8.65	8.65	1.63	4.61
11	- 2.99	3.46	- 10.19	10.19	4.18	6.99
12	- 9.06	9.06	4.59	6.21	- 10.37	10.37
13	- 17.64	17.64	7.53	7.53	- 19.40	19.40
14	- 22.25	22.25	- 9.15	13.68	- 25.17	25.17
15	- 26.53	26.53	- 17.18	17.18	- 20.79	20.79
16	- 25.52	25.52	- 14.54	14.54	- 19.29	19.29
17	- 17.19	17.19	- 5.20	7.74	- 29.27	29.27
18	- 17.74	17.74	- 2.88	10.09	- 17.32	17.32
19	- 15.88	15.88	- 19.62	19.62	- 14.15	16.28
20	3.13	7.31	- 12.42	12.42	- 11.70	11.70
21	- 5.87	5.87	- 2.94	4.99	- 13.03	13.03
22	- 22.69	22.69	- 9.19	9.19	- 9.09	9.09
23	- 11.03	11.03	- 3.76	9.83	- 28.02	28.02
24	- 9.73	10.05	16.09	21.43	- 36.25	36.25
25	16.82	16.82	- 25.08	25.08	- 26.66	26.66
26	13.19	13.19	- 32.75	32.75	- 11.73	11.73
27	14.41	14.41	- 30.59	30.59	- 10.72	10.72
28	19.19	19.19	- 25.60	25.60	- 8.74	9.64
29	34.75	34.75	- 8.43	12.26	- 6.90	6.90
30	37.33	37.33	- 22.54	22.54	- 4.44	5.36
31	35.49	35.49	- 8.72	10.35	- 11.95	11.95
32	74.34	74.34	10.16	10.70	- 2.15	5.57
33	83.64	83.64	0.86	5.92	19.11	19.11
34	84.31	84.31	4.47	6.50	21.79	21.79
35	56.65	56.65	- 0.31	7.01	26.65	25.65
36	61.62	61.62	- 26.90	26.90	38.45	38.45
37	44.31	44.31	.	.	41.60	41.60
38	.	.	.	.	38.93	38.93
39						

Table 5.3. (Cont.)

Weeks to Maturity	12-23-76		9-23-76		Over all Contracts	
	D	|D|	D	|D|	D	|D|
1	- 3.19	3.19	0.29	0.50	0.33	3.41
2	2.95	4.11	- 1.10	1.74	- 1.38	5.50
3	- 1.19	4.31	- 4.91	4.91	- 4.41	8.64
4	- 7.59	7.59	- 8.33	8.33	- 11.27	13.59
5	- 2.21	3.47	- 4.45	4.45	- 13.41	16.61
6	- 8.34	8.34	- 3.56	3.56	- 17.48	17.52
7	- 7.29	7.29	- 3.73	4.95	- 16.21	16.65
8	- 12.00	12.00	- 0.33	2.98	- 20.32	20.87
9	- 16.99	16.99	- 3.63	4.27	- 21.74	22.60
10	- 14.21	14.21	0.48	5.05	- 20.82	22.16
11	- 17.46	17.46	- 0.40	1.93	- 14.57	16.38
12	- 17.31	17.31	- 6.57	6.73	- 21.54	22.40
13	- 15.08	15.08	- 6.55	6.55	- 27.88	29.02
14	- 26.90	26.90	- 19.14	19.14	- 41.97	42.29
15	- 27.28	27.28	- 16.26	16.26	- 49.18	49.18
16	- 29.85	29.85	- 18.20	18.20	- 50.72	50.72
17	- 34.17	34.17	- 16.09	16.09	- 50.63	50.84
18	- 36.90	36.90	- 18.03	18.03	- 49.95	50.38
19	- 28.50	28.50	- 7.34	8.00	- 52.48	52.66
20	- 21.81	21.81	16.88	16.88	- 44.19	47.45
21	- 18.13	18.13	14.13	14.13	- 47.17	49.40
22	- 19.02	19.02	5.70	5.70	- 45.27	46.09
23	- 18.78	18.78	1.65	4.83	- 42.49	43.33
24	- 12.98	12.98	- 2.77	5.02	- 42.27	45.30
25	- 14.61	14.61	0.45	3.24	- 46.03	46.03
26	- 4.86	6.76	3.83	7.28	- 45.53	48.51
27	- 2.76	2.76	5.76	5.98	- 41.99	44.92
28	8.90	8.90	6.76	6.76	- 42.59	47.63
29	5.88	5.88	10.63	10.63	- 41.44	49.38
30	14.52	14.52	13.71	15.76	- 33.49	44.49
31	22.18	22.18	- 8.00	9.83	- 34.91	44.86
32	24.90	24.90	- 1.41	4.72	- 26.57	47.41
33	41.28	41.28	- 9.88	9.88	- 21.24	49.39
34	33.44	33.44	- 42.53	42.53	- 18.31	47.00
35	17.51	17.51	- 61.07	61.07	- 20.69	39.96
36	14.40	14.40	- 73.41	73.41	- 21.13	42.31
37	18.43	18.43	- 60.51	60.51	- 14.45	38.74
38	15.28	15.28	.	.	5.68	30.88
39						

The tables and charts presented above lead to the conclusion that the difference between the futures and forward rate is equal to zero only at the maturity of the futures contract and is significantly different from zero in the weeks preceding maturity. In addition, it is clear that over all contracts, the difference between the futures and forward rates follows a quadratic pattern over time. Having demonstrated the significance of the difference from zero over time, the question still remains as to whether this implies market inefficiency or not. If the difference can be explained on the basis of the variables which distinguish a futures from a forward contract, then it does not necessarily follow that a significant difference implies market inefficiency.

Regression Analysis

The objective of the regression analysis was to determine if transactions costs and the premium for risk were significant in explaining the difference over time. According to the theory, these variables, if they are significant, should provide the basis for believing that the difference between the futures and forward rates may not equal zero, even in an efficient market.

Transactions costs

In Chapter IV, it was assumed that the initial margin requirement ($1500) was satisfied by depositing Treasury securities with the broker or clearing house. Thus, the only transactions costs incurred were the ($60.00) commissions per contract. Paid at the termination of the contract, commissions represent a fixed cost with respect to the holding period. Therefore, it was assumed that they would have a declining influence the longer the holding period. Expressed as a percentage of the initial margin requirement, the variable used to represent the commission charges was:

$$C = \left(\frac{360}{DTM}\right)\left(\frac{60}{1500}\right), \qquad (5.20)$$

where C represents costs and DTM is, as before, days to maturity and $\partial C/\partial DTM < 0$.

The risk premium

A risk averter will demand a positive risk premium; those who prefer risk may require a negative risk premium. The premium developed in Chapter IV was based on the risk associated with the margin account, i.e., the risk of margin interest foregone, and the potential inability to meet margin calls when the broker or clearing house requires them. The components of the risk premium, the subjective utility function, and the variance of the returns on the futures position, are unobservable ex ante to the execution of the futures transactions.

Two variables were used as proxies for risk--days to maturity, and a time series variance of the observed futures rate. Days-to-maturity was used (intuitively) to capture the risk of the margin account as the length of the holding period increased. It was maintained that the risk associated with the margin account would increase with the length of the open futures position. The second proxy, the variance of the observed futures rate was calculated sequentially by days to maturity. Calculated for each day this proxy computed the variance on the observed futures rate by using the current observation and all previous observations. For example, the risk proxy at three days to maturity consisted of the variance of the futures rate calculated over the first three days; at four days to maturity, the first four rates were used, and so on until the last observation represented a variance over the whole time series of the futures rate.

Regression models and results

Over all contracts, the plot of the difference between the futures rate and the forward rate illustrates a quadratic pattern over time. Therefore, two regressions were conducted using the proxies in both linear and quadratic form. The specifications were as follows:

$$D = B_0 + B_1C + B_2DTM + B_3(DTM)^2 \qquad (5.21a)$$

and

$$D = B_0 + B_1C + B_2\sigma_{rf} + B_3\sigma^2_{rf} . \qquad (5.21b)$$

Table 5.4 shows the regression results for the model expressed in Equation 5.21a. The model was run once using the ordinary least squares (OLS) regression technique and then again using autoregression, specified by a first-order autoregressive process. Each row in Table 5.4 represents a separate futures regression. From left to

Table 5.4. Regression results of the first model: $D = B_0 + B_1C + B_2DTM + B_3(DTM)^2$

Contract	B_0	C	DTM	DTM^2	R^2	ρ
Overall	15.28 (7.58)*	-2.81 (-3.13)*N [a]	-0.84 (-27.29)*	0.003 (27.53)*	.84	-0.77 (-16.57)*
12-20-79	3.29 (0.30)	-2.79 (-0.69)	-1.12 (-6.64)*	0.003 (4.90)*N	.43	-0.859 (-22.24)*
9-20-79	10.7 (2.06)*	-0.65 (-0.29)	-0.75 (-9.08)*	0.002 (5.26)*	.73	-0.81 (-18.49)*
6-21-79	70.05 (8.93)*	-13.73 (-4.15)*N	-1.95 (-15.33)*	0.006 (12.28)*	.74	-0.91 (-29.09)*
3-22-79	27.51 (4.04)*N	-5.50 (-1.83)	-1.04 (-10.03)*	0.003 (6.77)*	.67	-0.94 (-36.76)*
12-21-78	45.81 (8.51)*	-8.71 (-3.74)*N	-1.79 (-21.63)*	0.006 (20.81)*	.77	-0.84 (-20.99)*
9-21-78	18.04 (3.30)*N	-1.07 (-0.32)	-1.19 (-14.64)*	0.005 (16.41)*	.69	-0.94 (-35.72)*
6-22-78	7.92 (2.46)*N	-0.54 (-0.40)	-0.67 (-12.82)*	0.002 (11.45)*	.64	-0.86 (-21.90)*
3-23-78	5.24 (1.64)	-0.68 (-0.49)	-0.75 (-15.18)*	0.003 (19.28)*	.78	-0.82 (-19.05)*
12-22-77	-7.18 (-1.86)	1.91 (1.14)	-0.58 (-9.79)*	0.003 (14.10)*	.74	-0.84 (-21.13)*
9-22-77	9.48 (2.03)*N	-1.44) (-0.71)	-0.60 (-8.13)*	0.003 (13.31)*	.78	-0.78 (-16.94)*
6-23-77	-7.17 (-1.17)	1.33 (0.74)	-0.04 (-0.60)	0.0001 (0.52)	.02	-0.75 (014.96)*
3-24-77	23.85 (7.33)*	-5.46 (-3.31)*	-0.72 (-14.44)*	0.003 (17.22)*	.69	-0.77 (-16.37)*

12-23-76	5.35	-0.86	-0.51	0.0002	.68	-0.75
	(1.67)	(-0.56)	(-9.51)*	(13.18)*		(-15.52)*
9-23-76	-28.15	6.42	0.55	-0.002	.37	-0.87
	(-5.41)	(2.85)*	(6.68)*	(-8.24)*		(-24.39)*

[a]Indication that the t-value becomes insignificant under a first-order autoregressive process.

*Indication of significance at the 5 percent level under ordinary least squares right, the columns include the intercept, B_0, transactions costs, C, days to maturity, DTM, the quadratic term in days to maturity, $(DTM)^2$, the coefficient of determination, R^2, and the coefficient of the first-order autoregressive process, ρ. Within the table are the coefficients on these variables below which, in parentheses, are the t-statistics which test their significance from zero. The major variables of interest are the coefficients and t-values corresponding to C, DTM, and $(DTM)^2$.

Over all contracts, the results show that C is significant at the 5 percent level under OLS, as signified by the asterisk beside the t value. However, as noted by "N", the t-value becomes insignificant at the 5 percent level when the model is run using an autoregressive process of the first order. The coefficient on the autoregressive process -0.77 is significant at the 5 percent level. These results are representative of all of the contracts. Of the 14 contracts, C is significant under OLS on only four contracts, 6-21-79, 12-21-78, 3-24-77, and 9-23-76. Even for these contracts, however, the coefficient on C becomes insignificant when the model is run using the autoregressive procedure. From these results, it is clear that commissions have little or nothing to do with the spread between the futures and forward rates.

Unlike commissions costs, the days-to-maturity variable is, with two exceptions, significant under both estimating procedures. Over all contracts, DTM has a coefficient of -0.84 and a t-statistic of

-27.29. With the exception of the 9-23-76 contract, where the t-statistic becomes insignificant under the autoregressive procedure, all coefficients on DTM are negative and significant. To capture the quadratic nature of the difference over time, DTM was raised to the second power and included in the analysis. The square of DTM increased rapidly and the coefficients on this variable were correspondingly small. With the exception of the 12-20-79 contract, each coefficient on $(DTM)^2$ was significant at the 5 percent level under OLS and autoregression. All coefficients were positive except for the 9-23-76 contract.

The results on DTM and $(DTM)^2$ show that the holding period of the contract is a significant variable in the explanation of the spread between the futures and the forward rates over time. As DTM increases, the difference becomes negative indicating that the forward rate rises relative to the futures. The opposite, however, is true with $(DTM)^2$. It is difficult to draw any strong implications regarding DTM and the nature of the risk premium. Because of the reversal in the signs on the two coefficients, little can be said about positive or negative risk aversion. The safest conclusion that can be rendered from the analysis is that if DTM is in fact a true proxy for the risk associated with the margin account, then it is apparent from the regression results that this type of risk is a significant variable in the explanation of the futures-forward difference.

The coefficient of determination for each regression was reported and all t-statistics on the coefficient of the first-order autoregressive process, ρ, were significant. In other words, the time series data had autocorrelated disturbances. This was the major reason why various t-values were inflated under OLS and became insignificant when the autoregressive procedure was used.

The results of the model expressed in Equation 5.21b are shown in Table 5.5, the format of which is similar to Table 5.4. As a proxy for risk, this model used the time-series variance on the futures rate. With regard to C, the coefficients are significant only over all contracts and for the 6-21-79 contract; otherwise, they are insignificant at the 5 percent level.

Over all contracts, neither σ_{r^f} nor $\sigma^2_{r^f}$ were significant under OLS at the 5 percent level. For each of the eight most recent contracts, the coefficients that were significant under OLS became insignificant when the variables were transformed under autoregression. Only four of the contracts had significant coefficients under OLS that did not become insignificant under autoregression: 1 -22-77, 9-22-77, 3-24-77, and 12-23-76. The coefficients that were significant were consistent in terms of their signs with the results of Table 5.4. The coefficients of determination were low and the autoregressive parameters were all significant.

None of the variables used in the second model is consistently significant. Commissions costs are insignificant, and as a measure of risk, the time-series variance is also unsignificant. The standard deviation and variance of the futures rate have little explanatory

Table 5.5. Regression results of the second model: $D = B_0 + B_1 C + B_2 \sigma_{rf} + B_3 \sigma^2_{rf}$

Contract	B_0	C	σ_{rf}	σ^2_{rf}	R^2	p
Overall	-39.36 (-3.07)*	11.96 (4.79)*	-4.21 (-0.31)N[a]	5.05 (1.54)	.37	-0.88 (-25.07)*
12-20-79	-44.91 (-2.05)*	6.73 (1.23)	4.49 (0.99)	-0.66 (-3.00)*N	.59	-0.81 (-18.03)*
9-20-79	29.60 (1.34)	-0.53 (-0.10)	-21.84 (-2.24)*	0.87 (0.77)	.39	-0.91 (-29.77)*
6-21-79	-774.99 (-11.13)*	108.96 (13.02)*	481.69 (8.47)*N	-78.78 (-6.64)*N	.70	-.78 (-16.34)*
3-22-79	-0.87 (-0.10)	5.03 (1.45)	-31.03 (-5.33)*N	2.68 (3.67)*N	.50	-0.95 (-40.89)*
12-21-78	12.38 (0.83)	-1.18 (-0.30)	-14.12 (-1.78)	-0.13 (-0.13)	.56	-0.91 (-28.62)*
9-21-78	207.45 (5.33)*	-35.72 (-3.84)*N	-194.26 (-6.01)*	37.78 (5.60)*N	.32	-0.94 (-37.57)*
6-22-78	33.59 (3.82)*	-3.41 (-1.57)	-99.70 (-6.87)*N	33.90 (6.01)*N	.47	-0.86 (-22.16)*
3-23-78	23.05 (0.50)	-5.17 (-0.65)	-25.93 (-0.44)	1.33 (0.07)	.06	-0.95 (-45.16)*
12-22-77	78.54 (5.48)*N	-12.27 (-3.64)*	-156.65 (-9.37)*	53.88 (11.06)*	.53	-0.83 (-20.03*)

9-22-77	5.72 (0.58)	-0.13 (-0.05)	-49.76 (-5.29)*	24.39 (10.45)*	.72	-0.85 (-21.33)*
6-23-77	-8.92 (-1.61)	1.75 (0.88)	-0.13 (-0.02)	-0.13 (-0.08)	.01	-0.75 (-14.98)*
3-24-77	10.01 (3.21)*N	-1.03 (-0.79)	-17.15 (-9.68)*	2.83 (13.68)*	.72	-0.74 (-14.55)*
12-23-76	15.62 (4.39)*N	-2.82 (-2.13)*N	-16.73 (-12.55)*	1.88 (16.91)*	.80	-0.65 (-11.55)*
9-23-76	1.42 (0.22)	0.17 (0.06)	-6.67 (-1.36)	0.84 (1.14)	.03	-0.91 (-28.81)*

[a]Indication that the t-value becomes insignificant under a first-order autoregressive process.

*Indication of significance at the 5 percent level under ordinary least squares.

power with respect to the difference between the futures rates and the forward rates over time. From these results, littl_ can be concluded regarding observed difference in the two rates over time.

Summary descriptive statistics and regression analysis

The objective of the empirical analysis was twofold: to test for the significance of the difference between the futures rate and the forward rate over time and secondly, to test for the significance of transactions costs and a risk premium as variables which explain the observed difference over time.

The results clearly indicated that the difference between the two rates over time was significantly different from zero. Although the difference at the time of maturity was close to zero, it became negative as the days to maturity increased. The cost of commissions did not enter as a significant variable under either model or type of procedure. Although, as a measure of risk, days to maturity was a highly significant variable in the model for explaining the difference, the time-series variance was not.

CHAPTER VI. SUMMARY AND CONCLUSIONS

Summary

This study was initiated to examine the structure of prices existing between the 90-day Treasury bill futures market and the cash market for Treasury bills. Questions concerning the value of information produced in the futures market, the effects of futures trading on the cash market, and market efficiency provided the initial stimulus for the inquiry. The study included three major objectives. The first was to construct a theoretical model of the equilibrium futures price based on arbitrage relationships between the cash and futures markets. Based on the theoretical analysis, the second objective involved the derivation of logical conclusions and testable hypotheses concerning the structure of futures and cash prices. Specific hypotheses regarding the difference between the futures rate and the forward rate implied in the Treasury bill yield curve were presented. To satisfy the third objective, standard statistical techniques were used to test for the validity of the behavioral relationships derived from the model.

Conceptual model

Composed of two parts, the emphasis of the first part of the analysis centered on the (arbitrage) development of the equilibrium futures price, given existing cash prices. An expression for the equilibrium futures price, FP*, was derived under two conditions--one where transactions costs were ignored and the other where they were included. The derivation showed that when transactions costs were included in the model, the equilibrium futures price was surrounded by a band of prices within which arbitrage was not profitable.

In the second part of the analysis, attention was given to the major differences which distinguish a futures contract from a forward contract. For example, it was shown that the daily resettlement process of the margin account was unique to futures markets and introduced an element of risk not present in forward transactions. Ignored by previous authors, the uncertainty generated by the margin account was incorporated into the conceptual framework using an expected utility approach. As a result, it was found that the observed futures rate included a premium for risk.

With the inclusion of risk and transactions costs in the model, the structure of yields between the futures and cash markets was re-evaluated. Three specific hypotheses were derived from the model. The first stated that the observed futures rate was composed of three components--the risk-free futures rate, transactions costs, and a risk premium. It was maintained that the forward rate was similar only to the risk free part of the futures rate. To test the first hypothesis, the difference between the futures rate and the forward rate was tested for its significance from zero. The second and third hypotheses postulated that transactions costs and the risk premium were significant variables in the explanation of the difference over time. These hypotheses were analyzed using regression techniques and the coefficients on transactions costs and the risk variables were tested for their significance from zero.

Empirical analysis

The empirical work analyzed 14 contracts which traded during the period from January, 1976, to December 1979. The data set consisted of 2463 observations on the futures and forward rates, the difference between them, days to maturity, and a time series variance of the observed futures rate.

The first part of the statistical analysis consisted of summary statistics which analyzed the difference between the futures rate and the forward rate by contract, over the average of all contracts and over time. It was found that at the maturity date of the futures contract, the two rates were identical but diverged as the days prior to maturity of the contract increased. Forward contracts traded at a premium relative to futures, and over time, the difference (futures rates minus forward rates), averaged approximately -30 basis points and exhibited a quadratic pattern as days to maturity increased. The difference was tested for its significance from zero by contracts and over time. The test statistics clearly indicated that the difference between the futures and the forward rates was significantly different from zero. This was the conclusion that was expected because it was hypothesized that the existence of transactions costs and risk premiums would cause a divergence between the futures rate and the forward rate.

The conclusions based on the summary statistics led to the second part of the empirical work, which consisted of regression analyses explaining the pattern of the difference over time. According to the second and third hypotheses of the model, transactions costs and a risk premium were used in regression models of the difference between futures and forward rates. Transactions costs were represented by commissions charges incurred in the futures market and two proxies for risk were used, in two separate models. The data used were time series and each specification of the regression models was analyzed using ordinary least squares regression techniques and autoregressive procedures. The coefficients on the independent variables were tested for their significance from zero. It was found that the fixed costs of commissions were not significant in the explanation of the difference. One of the proxies for the risk premium, days to maturity, was found to be highly significant; however, the second proxy, a time-series variance of the observed futures rate, was insignificant. The results showed that as a measure for the risk associated with the futures margin account, days-to-maturity is a significant variable which explains the difference between the observed futures rate and its risk-free counterpart, the forward rate.

General Conclusions

According to the theoretical model presented in this study, commissions costs and risk represent two variables which are significant to the analysis of the price structure existing between the futures market and the cash market for 90-day Treasury bills. The empirical work confirms the significance of the risk premium but discounts the importance of transactions costs. Based on the theoretical and empirical findings, it must be concluded that risk is a significant variable that affects the value of the observed futures rate vis a vis the forward rate. It must be further concluded that price analysis of the futures market should include the risk premium as a significant variable. For example, tests of market efficiency which analyze only the futures and forward rates and which ignore the role played by uncertainty are likely to be biased in their results. This study suggests that, on the basis of uncertainty, there is reason to believe that even in an efficient market, the futures rate may be significantly different from the forward rate.

The empirical results clearly indicate that futures prices are directly related to prices in the cash market for Treasury bills. The link is established due to the potential for arbitrage between the two markets. From this, it can be concluded that a relationship exists whereby the effects of futures trading will be transmitted to the cash market and vice versa. With respect to the concerns of the U.S. Treasury and the Federal Reserve system, the question then becomes, does futures market activity have a stabilizing or destabilizing effect on the spot market. To answer this, the price effects of speculation and hedging need to be analyzed.

The relationship between the futures rate, forward rate, and the risk premium seems to suggest that even though the futures and forward rates are not identical, they are consistent. The information produced in the futures market and embodied in the observed futures price seems to reflect similar information provided by the forward rates implied in the cash market. The consistency between these two sources of expectations of future short-term interest rates is important because futures market information is readily available at a very low cost, whereas the calculation of forward rate expectations is time-consuming and expensive. With respect to the value of information produced by the futures market, further analysis needs to be conducted to determine whether or not the futures rates provide unbiased estimates of short-term rates expected to prevail in the future.

Future Research Needs

The conclusions of this study are tempered by various debilities of the analysis. For example, the equilibrium price of a futures contract was determined on the basis of partial equilibrium analysis, given cash prices. In a disequilibrium situation, it was the futures price, not the cash price, that changed. This fact led to the conclusion that individual investors expected a net interest gain in their margin accounts. However, in disequilibrium situations, it may be the cash prices that change, instead of the futures price. And perhaps, even more likely, both cash and futures prices may change simultaneously in the movement towards a dynamic, two-market equilibrium. These considerations do not change the basic conclusion that risk is an important component in the observed futures price. However, the analysis of the static and dynamic relationship between futures and cash prices might be better served on the basis of a general equilibrium analysis which emphasizes the simultaneous determination of equilibrium cash and futures prices.

The empirical testing of behavioral relationships is often made difficult by the fact that many economic variables are had to quantify. In the present study, the major obstacle in the empirical analysis was the measurement of the risk premium. Of the two variables used as a proxy for the risk premium, only days-to-maturity was significant in the analysis. It might be argued that this variable represented no more than a trend in the time series analysis; however, the results of the autoregressive estimation procedures indicated that days-to-maturity was a significant explanatory variable. Although it was expected that the time-series variance of the observed futures rate would also be significant, it was not. This may have been the result of inadequate data. Because the data on the futures rate were limited to the daily settlement prices of the futures contracts, only an inter-day time-series could be calculated. Perhaps, a better proxy for risk would have been a cross-sectional variance based on price information on all intra-day trades. These data were unavailable. The challenge for future research is to derive a proxy for risk which more accurately represents the elements of the subjective utility function and the variance of the return on the futures position.

In addition to these considerations, it is quite possible that other kinds of risk in the futures and the forward market transactions are ignored in the preceding analysis. Especially with regard to forward transactions, which occur not on an organized exchange, but between individual parties, further research is needed to evaluate risk and its effect on the price structure of the market.

The intensive study of an economic problem provides the investigator with the capacity to evaluate the relative importance of various concepts. In this study, the analysis of the futures-cash price structure was based on arbitrage relationships between the two markets. Arbitrage, however, is only one aspect of the Treasury bill futures market. The study of hedging and speculation, if they can be so easily distinguished, is just as important as questions regarding market efficiency, the value of futures market information, or the effect of futures trading on the cash market.

The futures market for Treasury securities is a new and innovative tool for the management of interest rate risk. Many financial institutions are unaware of or unprepared to capitalize on the opportunities provided by this market. They are unfamiliar with futures markets and their role in reducing market risk. Research is needed to analyze hedging and risk management, basis relationships over time, market strategies, futures accounting, and other operational aspects of the market.

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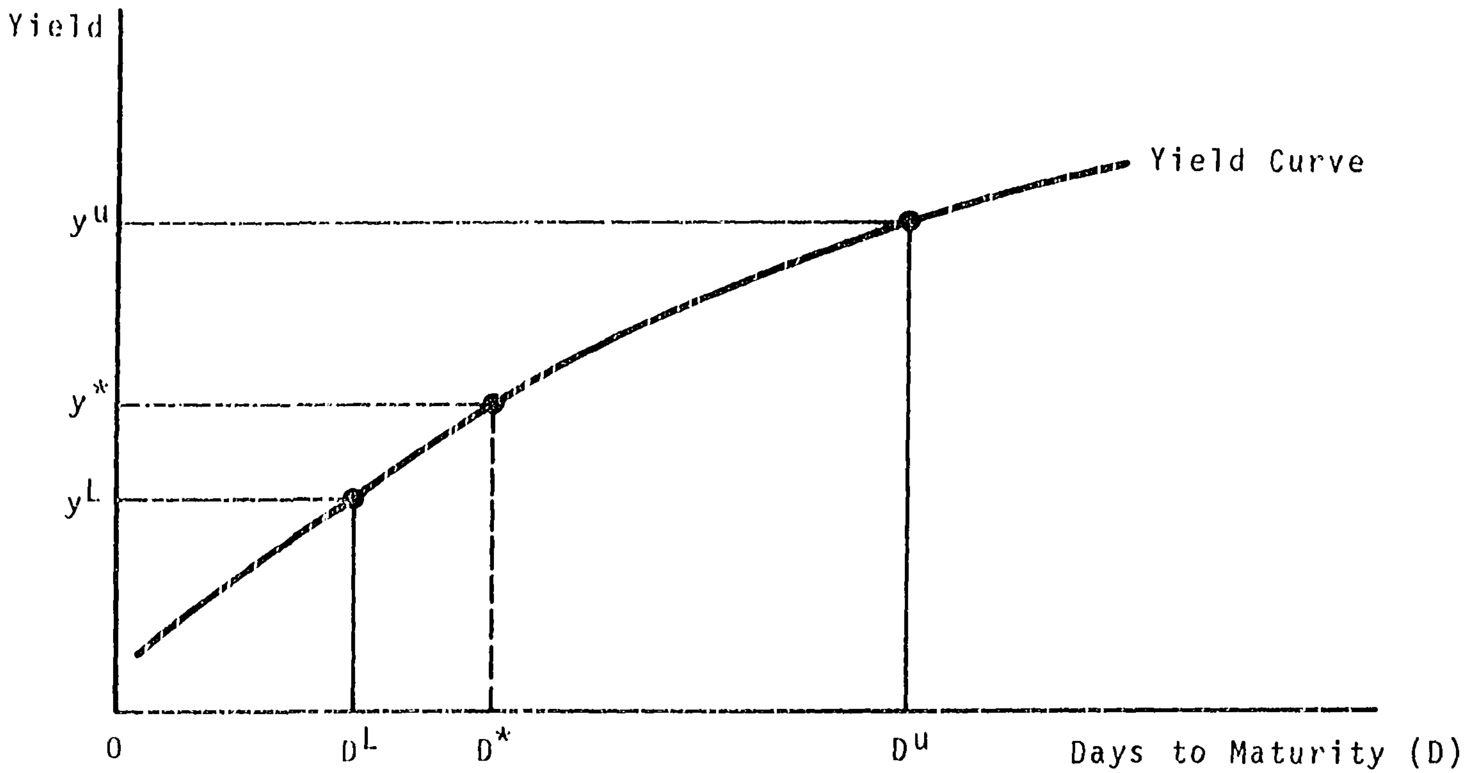
APPENDIX A: FUTURES CONTRACTS AND FORWARD RATE SPECIFICATIONS

Table A.1. Spot market treasury bills used in calculating forward rates

Futures Contract	Matched Bills		Interpolated Bills				Number of Observations		
			Nearby		Nearby + 3 months				
	Nearby	Nearby + 3 months	Lower	Upper	Lower	Upper	Matched	Mis-Matched	Total
12-20-79	12-20-79	3-20-80	12-11-79	1- 8-80	3- 4-80	4- 1-80	62	113	175
9-20-79	9-20-79	12-20-79	9-18-79	10-16-79	12-11-80	1- 8-80	63	114	177
6-21-79	6-21-79	9-20-79	5-29-79	6-26-79	9-18-79	10-16-79	62	106	168
3-22-79	3-22-79	6-21-79	3- 6-79	4- 3-79	5-29-79	6-26-79	62	120	182
12-21-78	12-21-78	3-22-79	12-12-78	1- 9-79	3- 6-79	4- 3-79	62	117	179
9-21-78	9-21-78	12-21-78	9-19-78	10-17-78	12-12-78	1- 9-79	64	112	176
6-22-78	6-22-78	9-21-78	5-30-78	6-27-78	9-19-78	10-17-78	63	105	168
3-23-78	3-23-78	6-22-78	3- 7-78	4- 4-78	5-30-78	6-27-78	61	115	176
12-22-77	12-22-77	3-23-78	12-13-77	1-10-78	3- 7-78	4- 4-78	62	117	179
9-22-77	9-22-77	12-22-77	9-20-77	10-18-77	12-13-77	1-10-78	63	115	178
6-23-77	6-23-77	9-22-77	5-31-77	6-28-77	9-20-77	10-18-77	64	106	170
3-24-77	3-24-77	6-23-77	3- 8-77	4- 5-77	5-31-77	6-28-77	64	116	180
12-23-76	12-23-76	3-24-77	12-14-76	1-11-77	3- 8-77	4- 5-77	62	119	181
9-23-76	9-23-76	12-23-76	9-21-76	10-19-76	12-14-76	1-11-77	64	114	178



General form for the linear interpolation of y*:

$$y^* = \frac{(D^u - D^*)\,(y^L) + (D^* - D^L)\,(y^u)}{(D^u - D^L)}$$

Figure A.1. The cash market yield curve and the general form for the linear interpolation of Y*

Table A.2. Equations for the linear interpolation of cash market yields (used to calculate implied forward rates), by contract

Contract	Equations for the linear interpolation of cash market yields corresponding to:[a] The maturity date of the futures contract	90 days following the maturity date of the futures contract
12-20-79	$Y_M = ((20 \times Y_{ML}) + (10 \times Y_{MU}))/29;$	$Y_N = ((13 \times Y_{NL}) + (17 \times Y_{NU}))/29$
9-19-79	$Y_M = ((27 \times Y_{ML}) + (3 \times Y_{MU}))/29;$	$Y_N = ((20 \times Y_{NL}) + (10 \times Y_{NU}))/29$
6-21-79	$Y_M = ((6 \times Y_{ML}) + (24 \times Y_{MU}))/29;$	$Y_N = ((27 \times Y_{NL}) + (3 \times Y_{NU}))/29$
3-22-79	$Y_M = ((13 \times Y_{ML}) + (17 \times Y_{MU}))/29;$	$Y_N = ((6 \times Y_{NL}) + (24 \times Y_{NU}))/29$
12-21-78	$Y_M = ((20 \times Y_{ML}) + (10 \times Y_{MU}))/29;$	$Y_N = ((13 \times Y_{NL}) + (17 \times Y_{NU}))/29$
9-21-78	$Y_M = ((27 \times Y_{ML}) + (3 \times Y_{MU}))/29;$	$Y_N = ((20 \times Y_{NL}) + (10 \times Y_{NU}))/29$
6-22-78	$Y_M = ((6 \times Y_{ML}) + (24 \times Y_{MU}))/29;$	$Y_N = ((27 \times Y_{NL}) + (3 \times Y_{NU}))/29$
3-23-78	$Y_M = ((13 \times Y_{ML}) + (17 \times Y_{MU}))/29;$	$Y_N = ((6 \times Y_{NL}) + (24 \times Y_{NU}))/29$
12-22-77	$Y_M = ((20 \times Y_{ML}) + (10 \times Y_{MU}))/29;$	$Y_N = ((13 \times Y_{NL}) + (17 \times Y_{NU}))/29$
9-22-77	$Y_M = ((27 \times Y_{ML}) + (3 \times Y_{MU}))/29;$	$Y_N = ((20 \times Y_{NL}) + (10 \times Y_{NU}))/29$
6-23-77	$Y_M = ((6 \times Y_{ML}) + (24 \times Y_{MU}))/29;$	$Y_N = ((27 \times Y_{NL}) + (3 \times Y_{NU}))/29$
3-24-77	$Y_M = ((13 \times Y_{ML}) + (17 \times Y_{MN}))/29;$	$Y_N = ((6 \times Y_{NL}) + (24 \times Y_{NU}))/29$
12-23-76	$Y_M + ((20 \times Y_{ML}) + (10 \times Y_{MN}))/29;$	$Y_N = ((13 \times Y_{NL}) + (17 \times Y_{NU}))/29$

9-23-76 $Y_M = ((27 \times Y_{ML}) + (3 \times Y_{MN}))/29$; $Y_N = ((20 \times Y_{NL}) + (10 \times Y_{NU}))/29$

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[a]Y_M = interpolated cash market yield corresponding to time M.

Y_{ML} = observed yield of cash bill maturing just prior to time M.

Y_{MU} = observed yield of cash bill maturing just after time M.

Y_N = interpolated cash market yield corresponding to time N.

Y_{NL} = observed yield of cash bill maturing just prior to time N.

Y_{NU} = observed yield of cash bill maturing just after time N.

APPENDIX B: GRAPHS OF THE DIFFERENCE BETWEEN FUTURES AND FORWARD RATES FOR EACH CONTRACT BY DAYS-TO-MATURITY

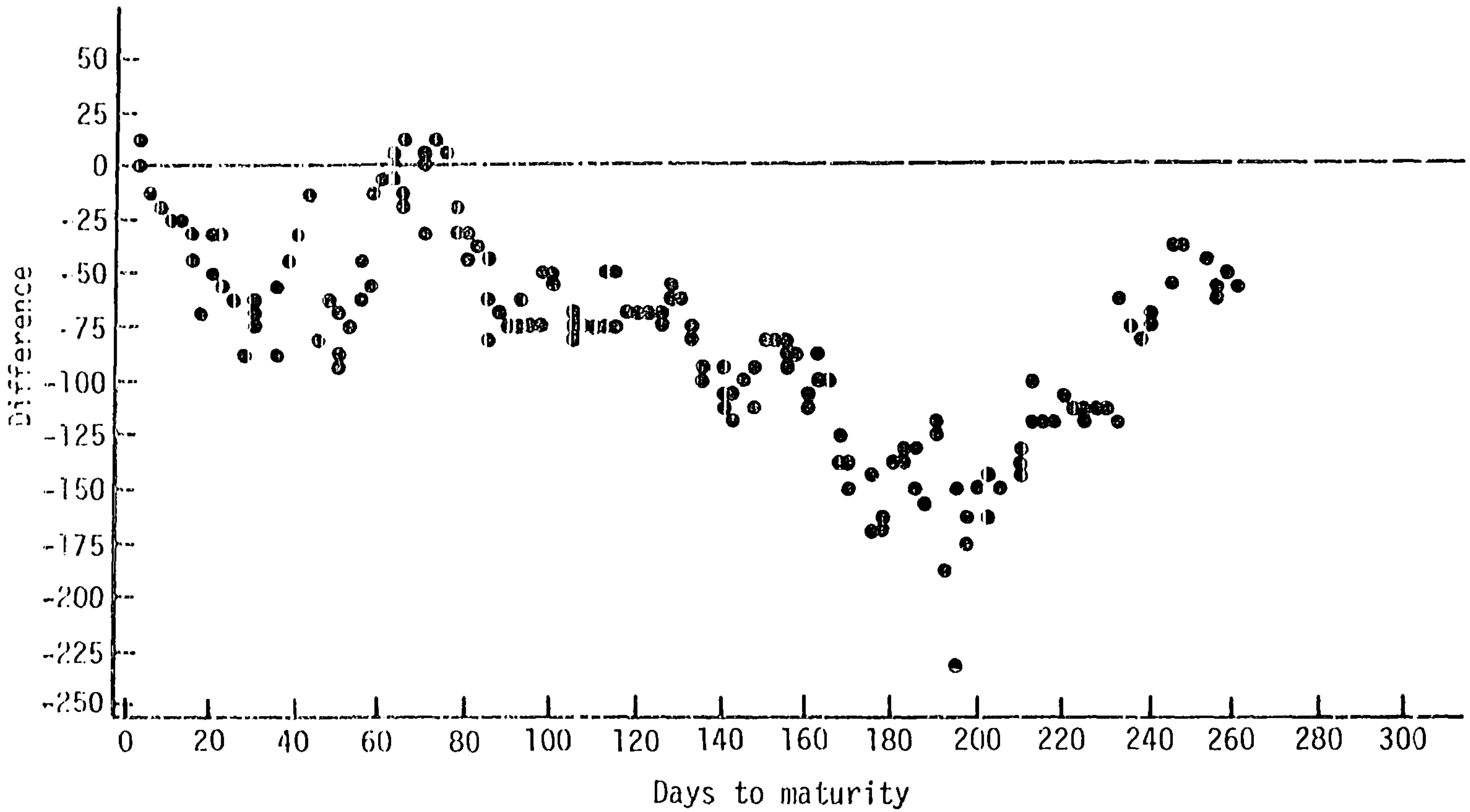

Figure B.1. Contract 12-20-79. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

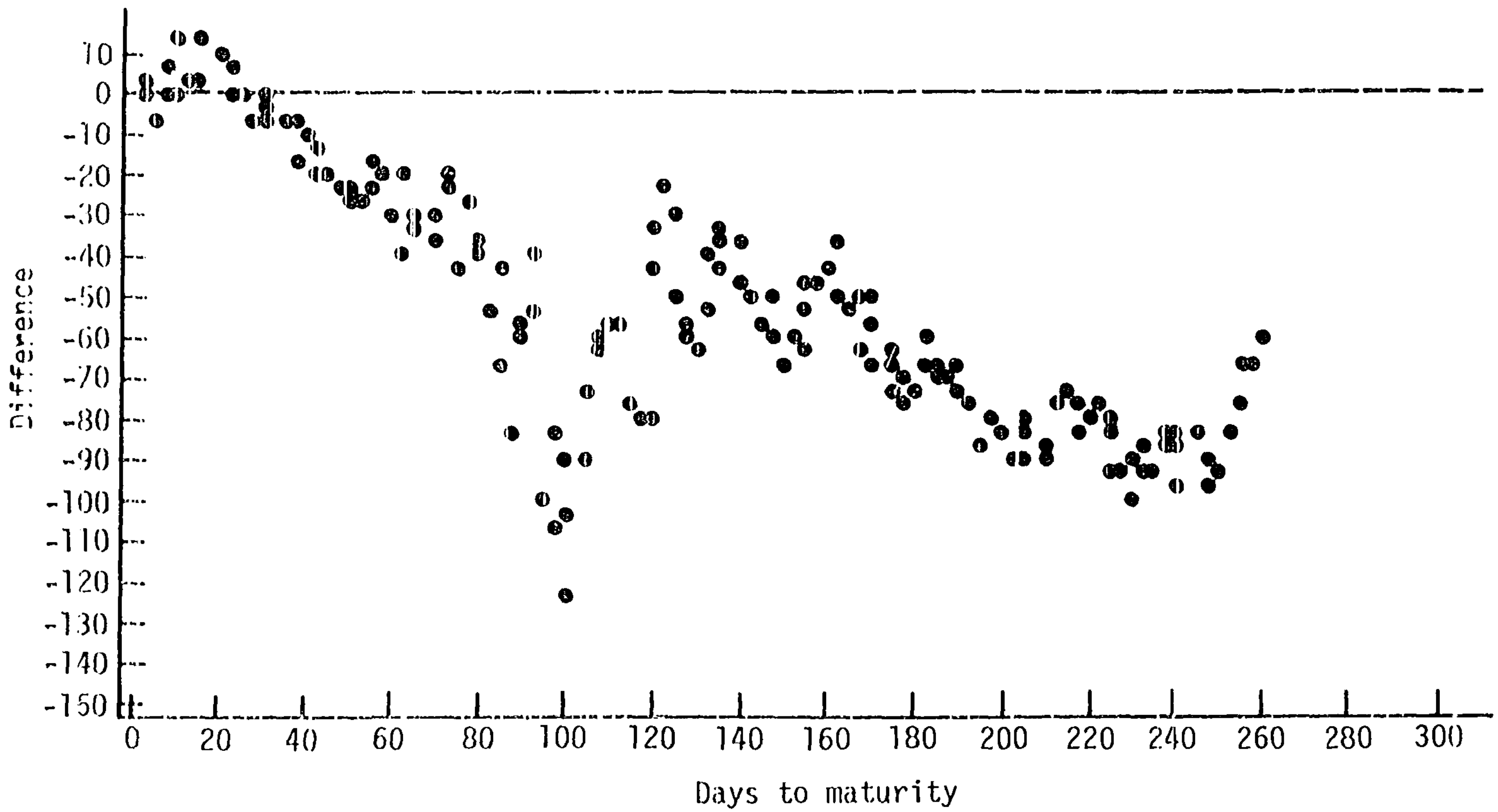

Figure B.2. Contract 9-22-79. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

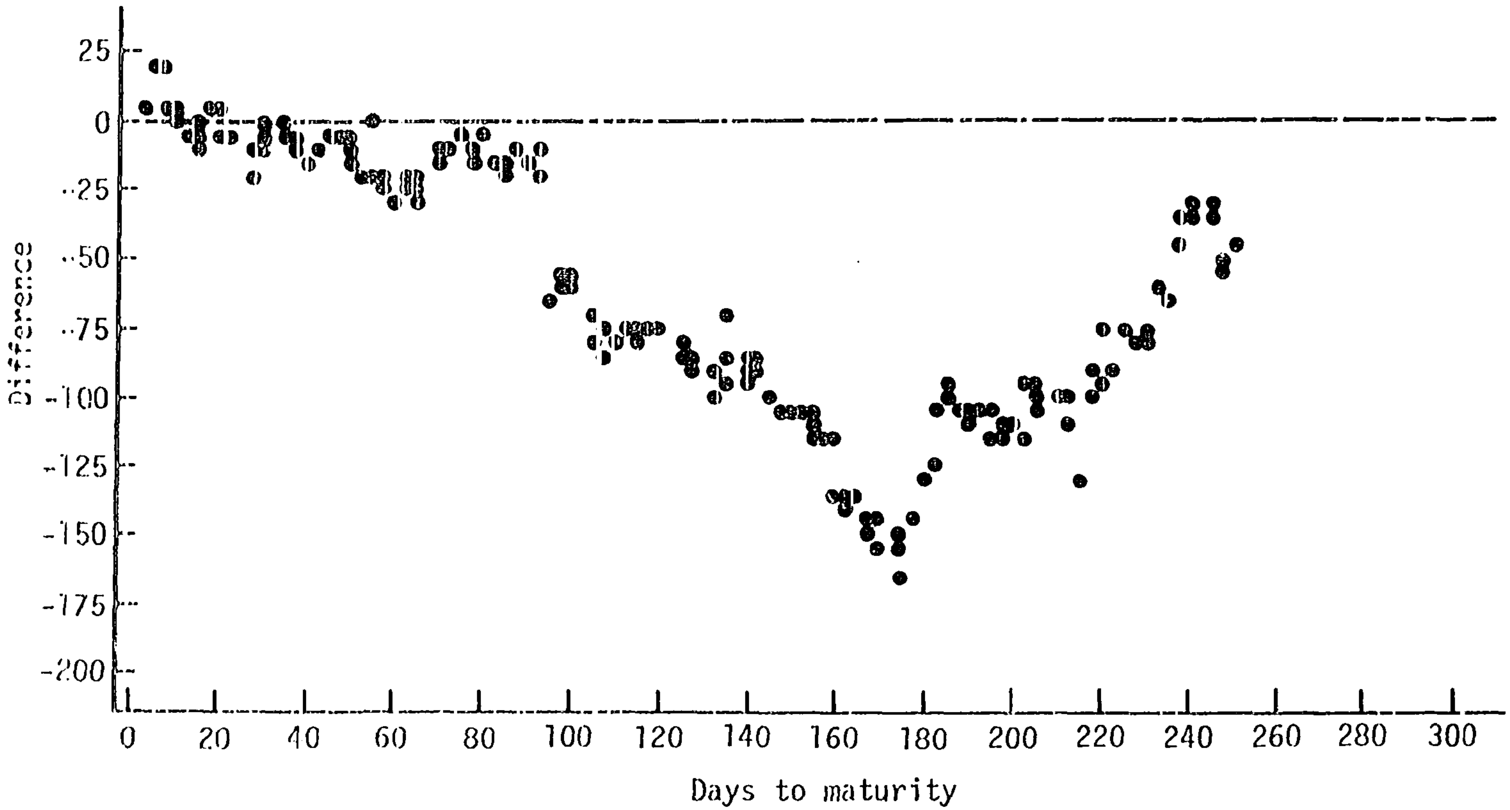

Figure B.3. Contract 6-21-79. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

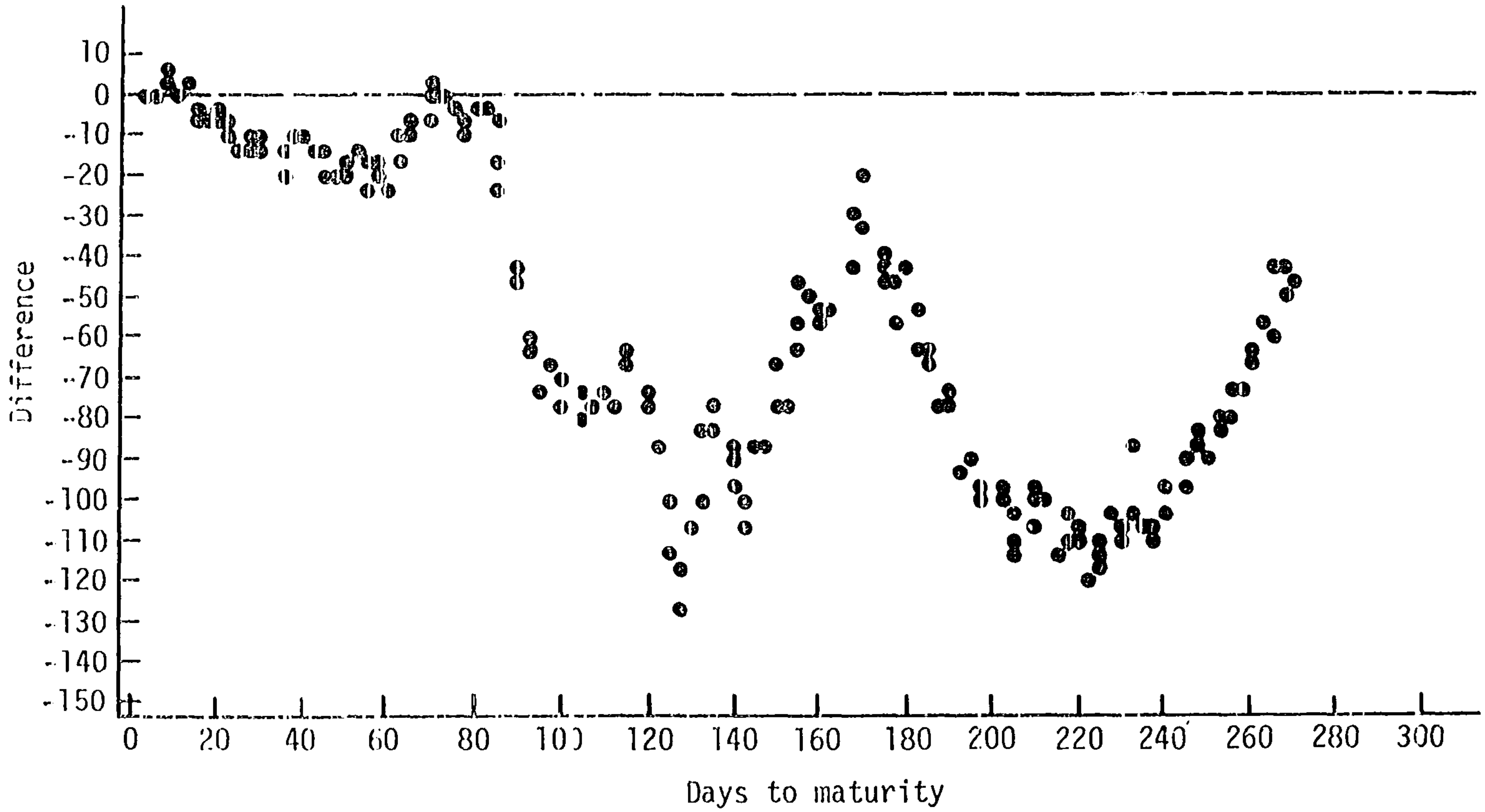

Figure B.4. Contract 3-22-79. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

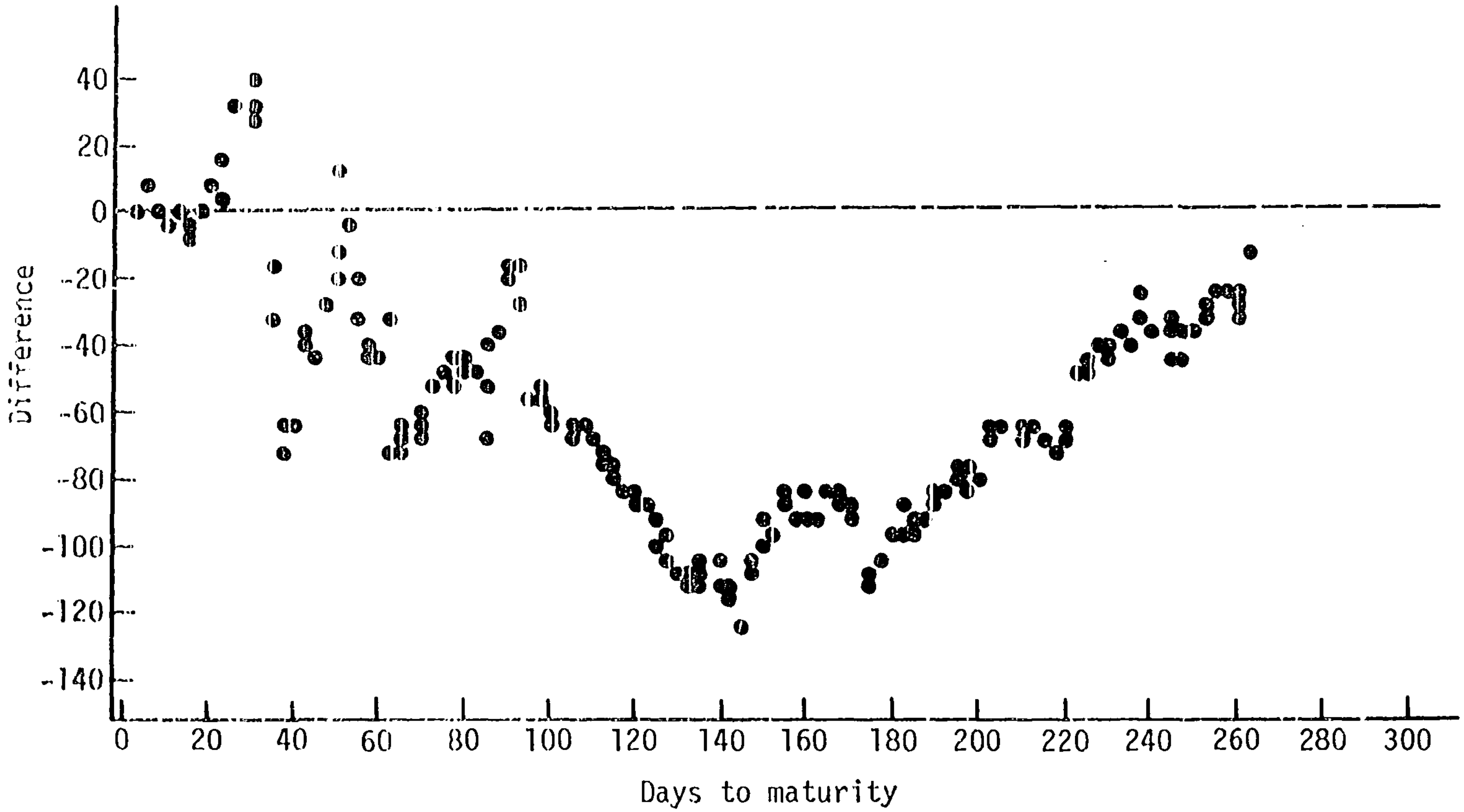

Figure B.5. Contract 12-21-78. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

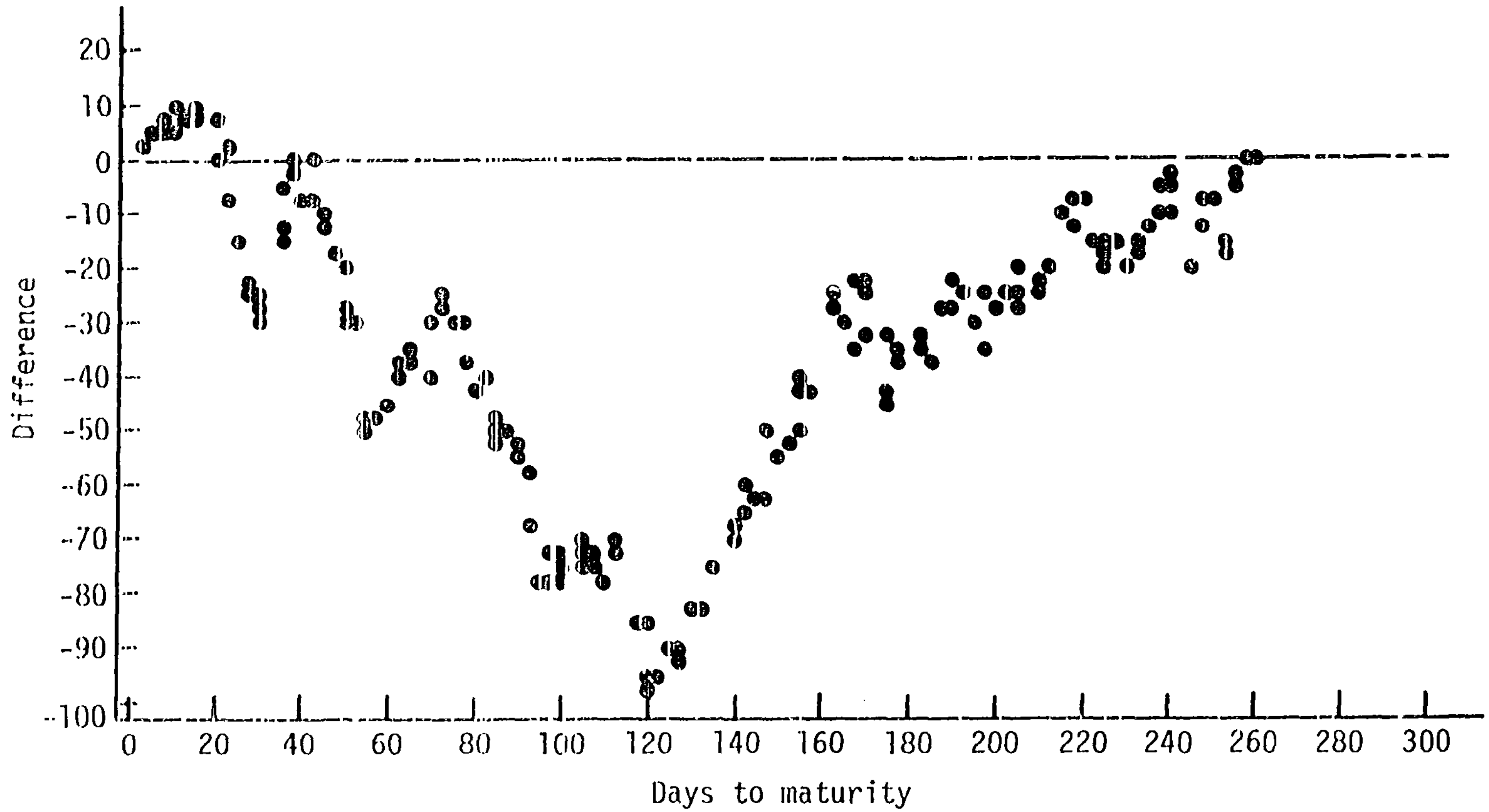

Figure B.6. Contract 9-21-78. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

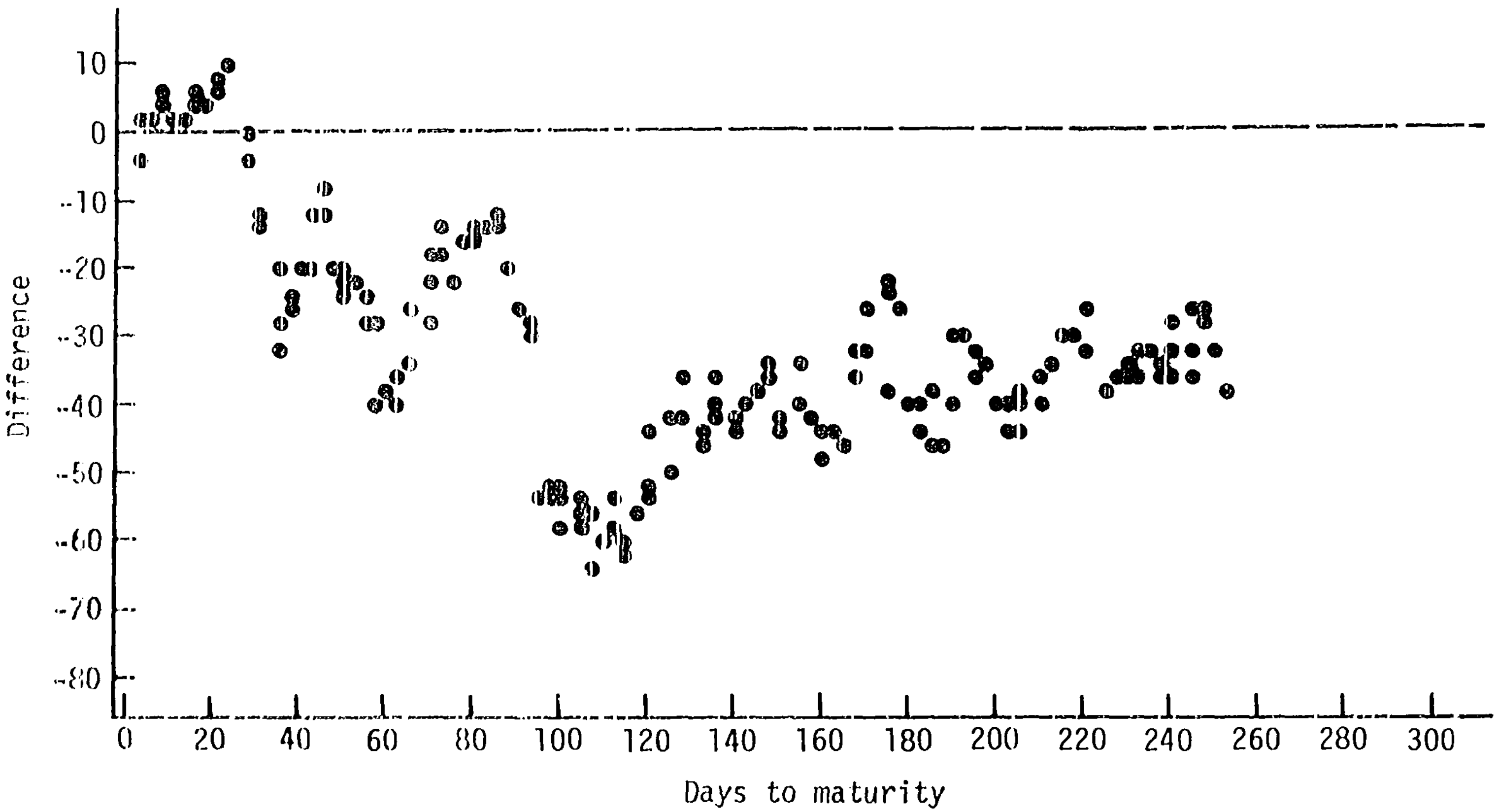

Figure B.7. Contract 6-22-78. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

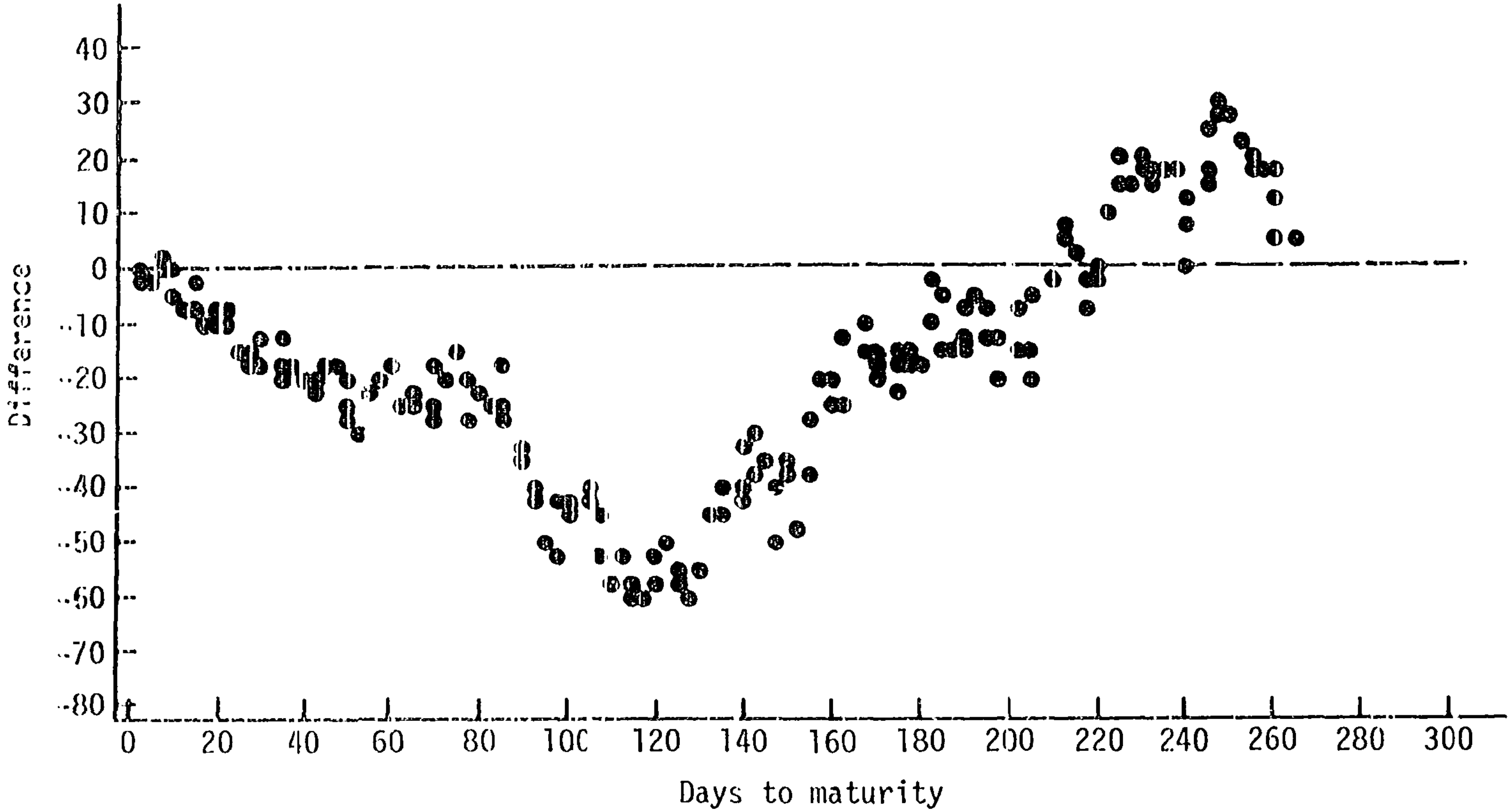

Figure B.8. Contract 3-23-78. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

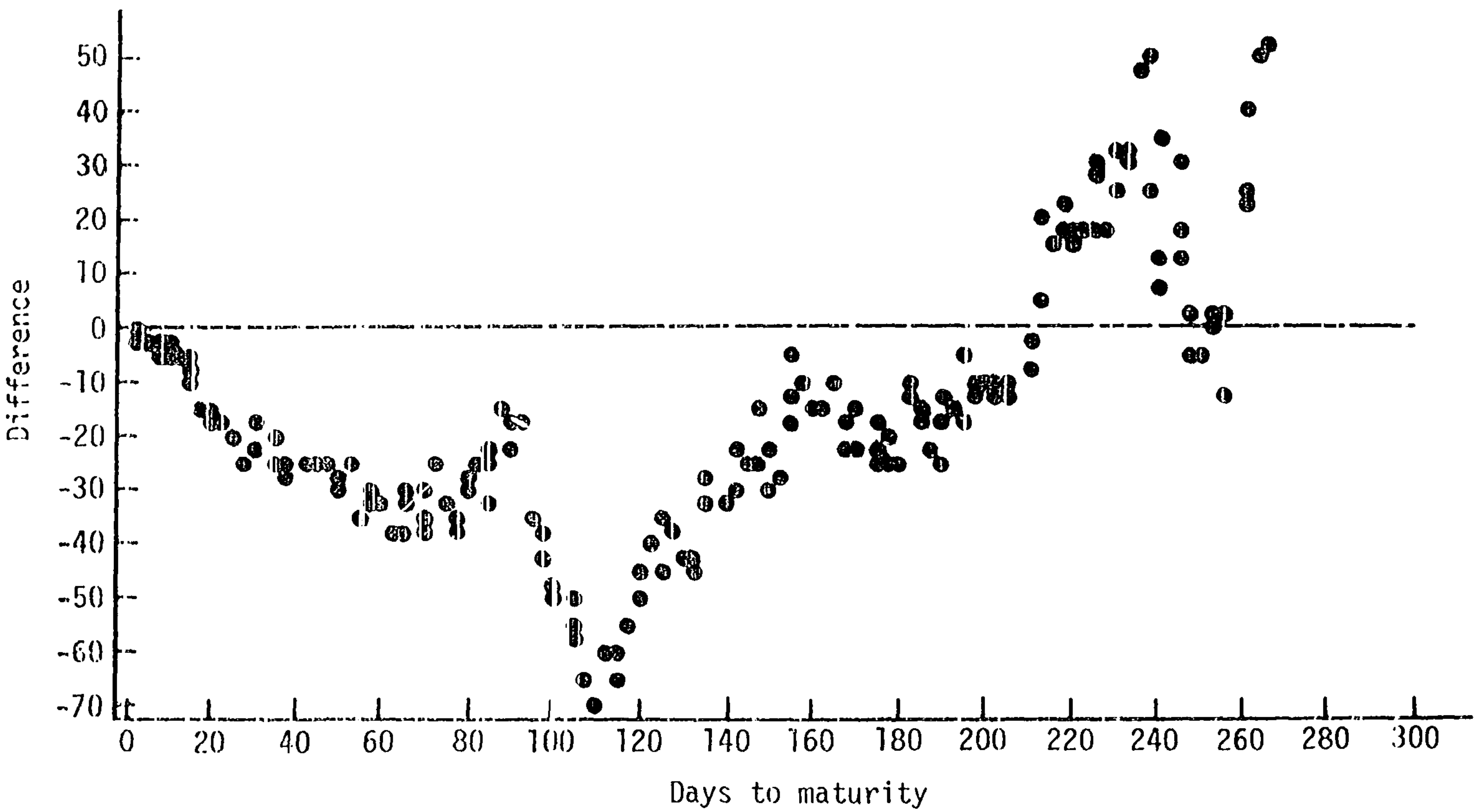

Figure B.9. Contract 12-22-77. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

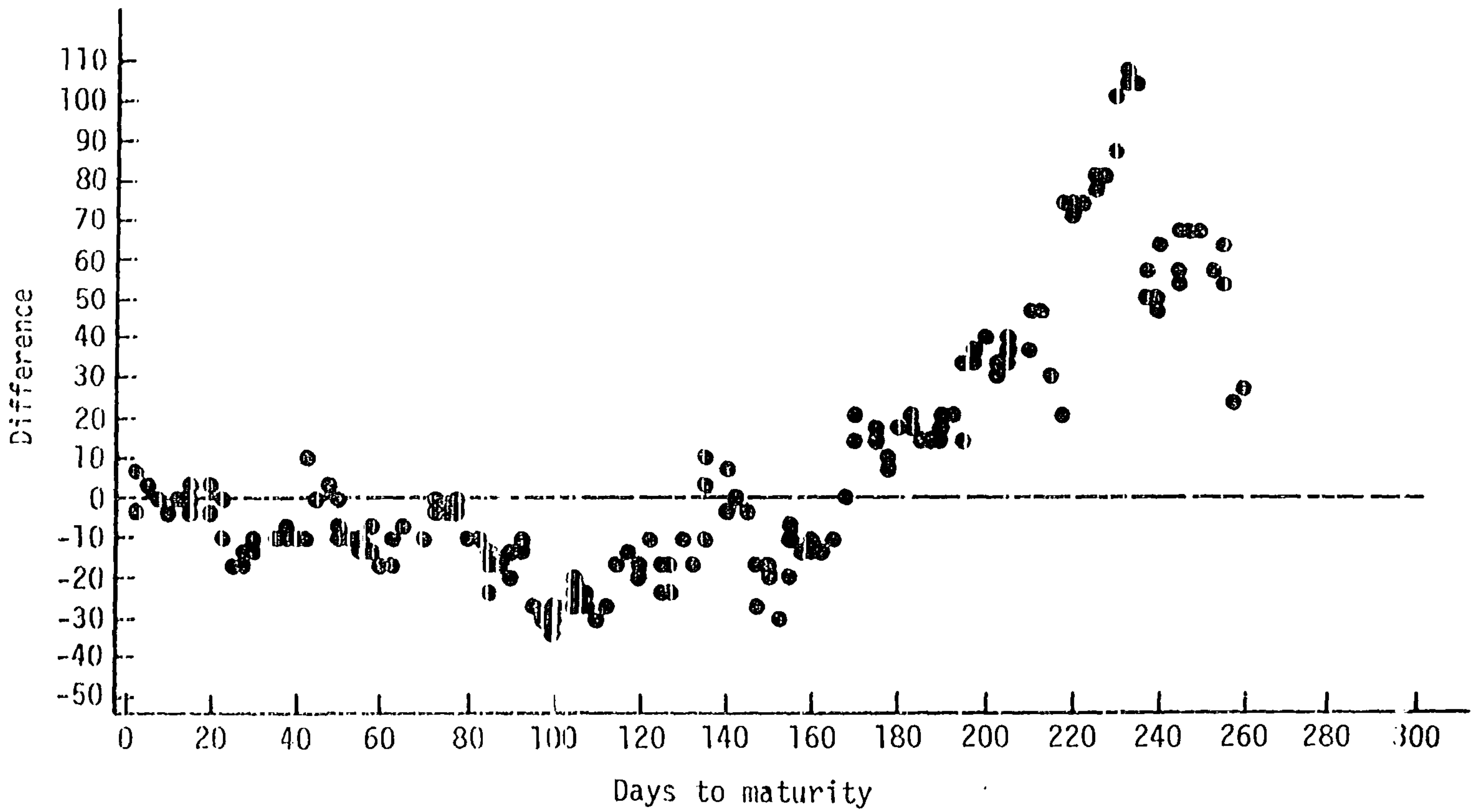

Figure B.10. Contract 9-22-77. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

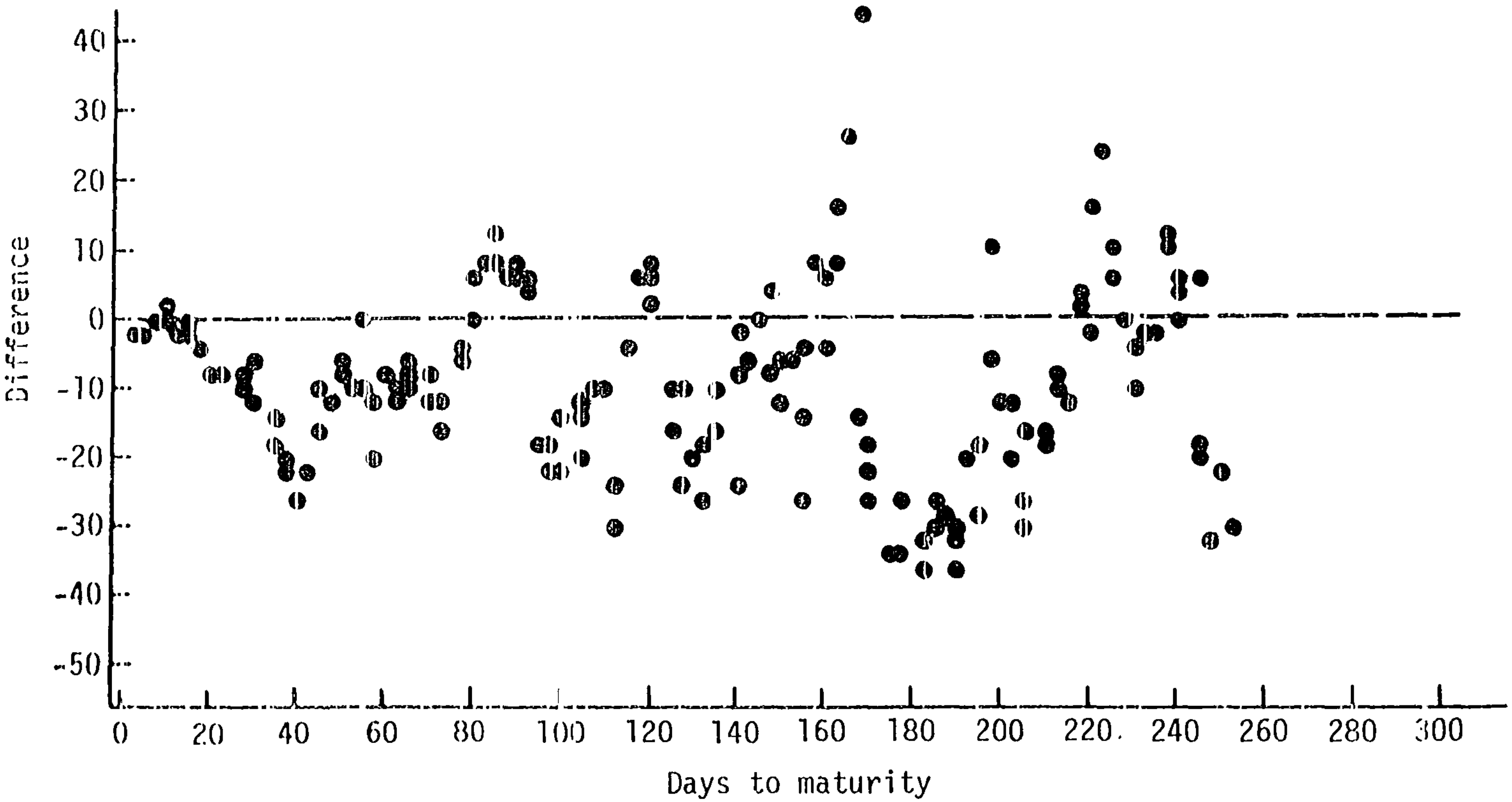

Figure B.11. Contract 6-23-77. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

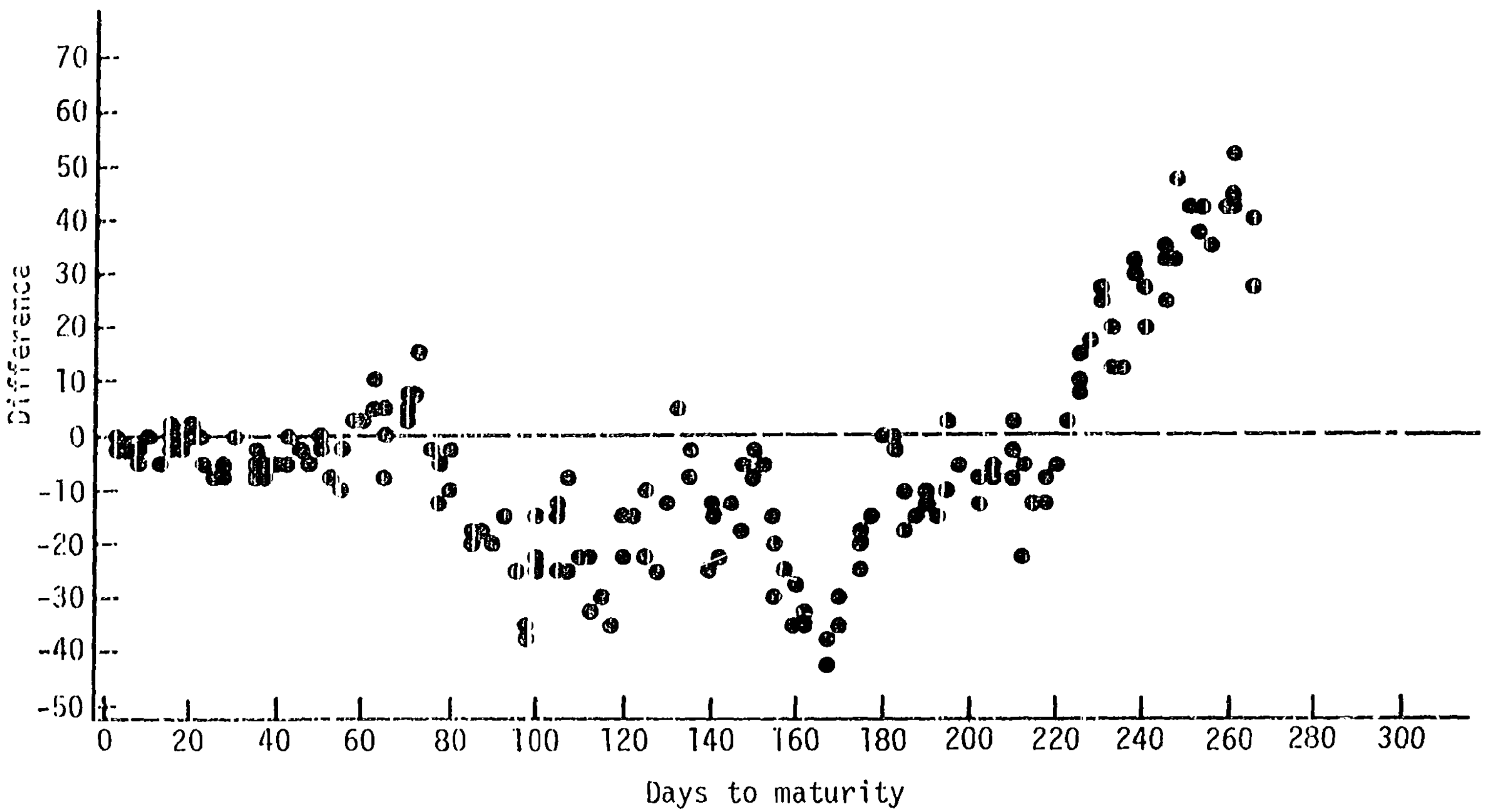

Figure B.12. Contract 3-24-77. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

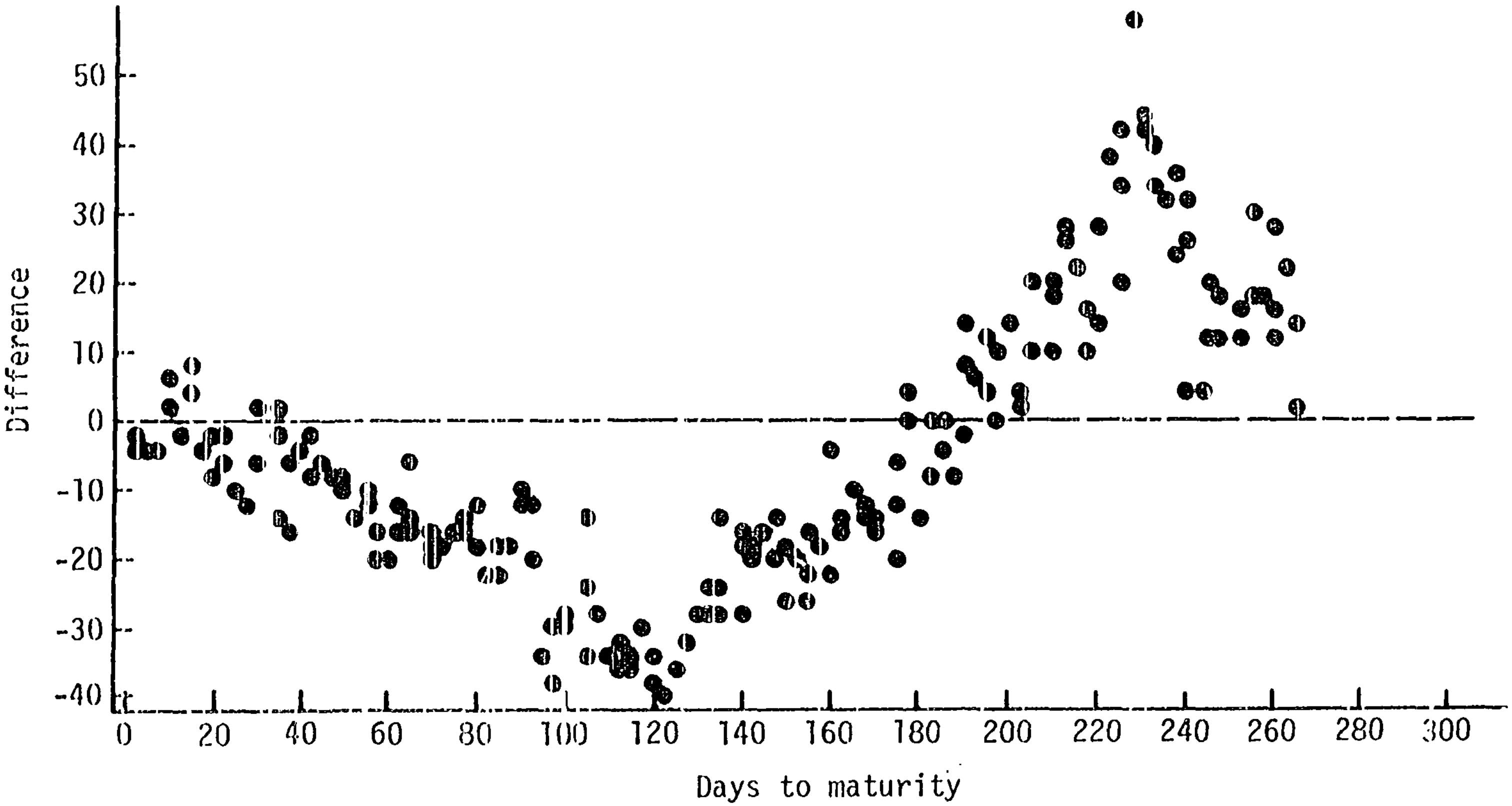

Figure B.13. Contract 12-23-76. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points

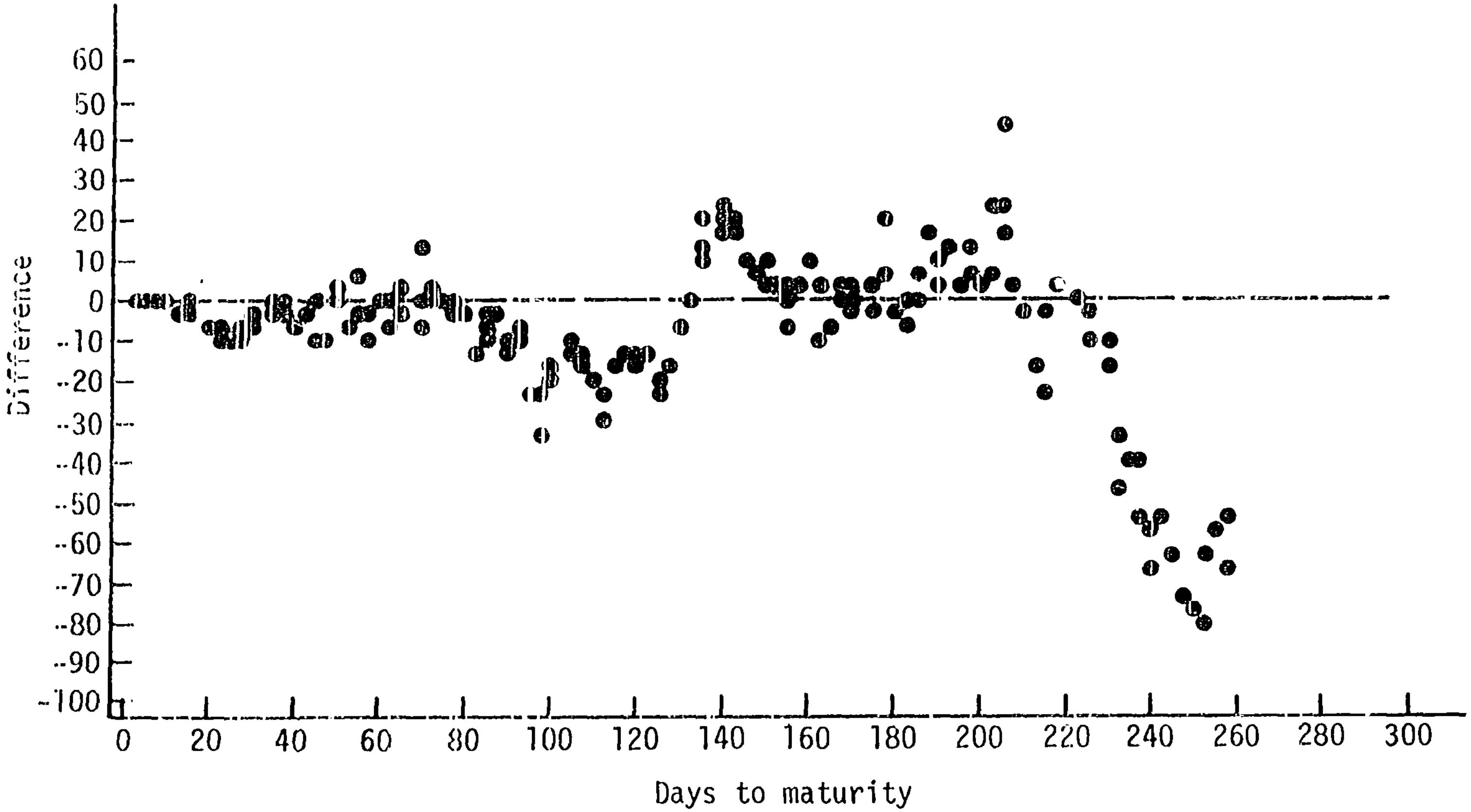

Figure B.14. Contract 9-23-76. Plot of the futures minus forward rate by days-to-maturity, expressed in basis points